Massachusetts

Department of Education

Massachusetts Curriculum Frameworks for Science and Technology

Review Draft

October 1994

NOTE: The attached material is still being discussed by the Massachusetts Department of Education and does not represent Massachusetts Department or Board of Education Policy.

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October 1994

Partnerships Advancing the Learning of Mathematics and Science
Massachusetts Department of Education
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Table of Contents

The Commonwealth of Massachusetts
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obert V. Antonucci
Commissioner

October, 1994

Dear Colleagues and Community Members:

The Massachusetts Department of Education recently completed drafts of the Curriculum Frameworks in Mathematics and in Science & Technology. These frameworks are guides that will be especially useful to district curriculum supervisors, and can also be used by teachers, schools, and institutions of higher education to assist in the planning and evaluation of programs. The frameworks implement and support the Common Core of Learning which identifies broad, measurable results for students.

Both documents have been prepared by two writing teams under the direction of PALMS, Partnerships Advancing the Learning of Mathematics and Science, our statewide initiative. The drafts were previewed by a panel of mathematics, science and Department professionals earlier this year. Revisions have been made based on their recommendations.

Although the frameworks are still under review, we are now releasing them for dissemination in your school district. I encourage you to reprint each framework so that copies are available for every building principal, and mathematics and science department heads. We will be notifying all teachers of this process.

Public responses to the Mathematics and Science & Technology Frameworks will be collected through January 9, 1995. We will conduct regional forums as well as focus groups to gather suggestions and build consensus. A survey form is also enclosed within the frameworks for your convenience.

I look forward to hearing your comments and especially look forward to continuing our work together as we strengthen public education in Massachusetts.

Sincerely,

Robert V. Antonucci

Robert V. Antonucci
Commissioner

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A response form for readers is included at the back of this document. Please return this form with your comments by January 9, 1995.

Preface

The Department of Education is pleased to present the drafts of the first two frameworks: Mathematics, and Science and Technology. With the development of the first state curriculum frameworks, Massachusetts embarks on a new era in educational improvement. The frameworks are designed as tools to be used by educators in planning instructional programs from pre-kindergarten through grade 12 and for adult basic education. Although school districts will not be required to adopt the Curriculum Frameworks, they may refer to the Frameworks as guidelines to bring about improvement in the education of all the students in the Commonwealth.

The Mathematics and the Science and Technology Frameworks are based upon two reform initiatives in Massachusetts: PALMS and the Education Reform Act of 1993.

PALMS, Partnerships Advancing the Learning of Mathematics and Science, is the Statewide Systemic Initiative, funded in 1993 by the National Science Foundation. **[Include something on history and/or goals of PALMS.]**

The Massachusetts Education Reform Act was enacted to strengthen a high quality public education system to all children in the state so they have the opportunity to reach their full potential and lead productive lives. A key component of the Education Reform Act is the Common Core of Learning which sets broad educational goals and is a first step in the education reform process. The Common Core of Learning has been adopted by the State Board of Education and is available from the Massachusetts Department of Education. The next step is the development of state curriculum frameworks for the areas of arts, English, foreign languages, health, history and social studies, mathematics, and science and technology. These frameworks will contain academic standards which will lend themselves to objective measurement. The third step is the development of a comprehensive assessment system which evaluates student performance and measures the success of schools.

The creation of the first two frameworks, Mathematics and Science and Technology, was a collaborative endeavor among the members of the Framework Development Committee—teachers, school and district administrators, scientists, mathematicians, college faculty, parents, and representatives of business and

Mathematics, Science & Technology:

Know and understand major mathematical concepts such as measurement, estimation, quantity, probability and statistics; and, explore the relationship of mathematics to other areas of knowledge;

Recognize and use patterns, construct mathematical models, represent and reason about quantities and shapes, draw accurate conclusions from data, and solve, justify and communicate solutions to problems;

Apply the fundamental principles of the life sciences, physical sciences, earth/space sciences and the science of technology to analyze problems and relate them to human concerns and life experiences;

Investigate and demonstrate methods of scientific inquiry and experimentation.

--Massachusetts Common Core of Learning, adopted July 1994

community organizations across the state. A majority of the members are currently classroom teachers who have extensive experience teaching mathematics and science at elementary, middle, and high school levels, in urban, suburban, and rural settings. Some teach bilingual classes and some teach vocational education.

The committee met monthly between July 1993 and May 1994 discussing and deliberating the topics that are addressed in these Frameworks. While there was one Framework Development Committee, the committee was composed of two working groups, one for mathematics and one for science and technology. This unique committee structure allowed both working groups to collaborate extensively on issues that are generic to both mathematics and science. This joint effort resulted in the development of three chapters that are common to both Frameworks. Chapter 1, an introductory chapter, identifies some key issues related to effective mathematics and science education. Chapter 2 describes the learning, teaching, and assessment of mathematics and science. Chapter 4 presents some of the issues that will affect the successful implementation of new mathematics and science education practices.

Chapter 3 in each Framework is specific to the content area. It addresses the mathematics or science that all students should know and be able to do. In the Mathematics Framework, Chapter 3 builds upon the *Curriculum and Evaluation Standards* developed by the National Council of Teachers of Mathematics (NCTM). Chapter 3 of the Science and Technology Framework is based upon the preliminary versions of the *National Science Education Standards* being developed by the National Research Council (NRC).

[Include a concluding paragraph]

A Vision of Mathematics and Science Education

Mathematics and science as academic disciplines and tools for problem solving are central to the vitality of the economy and quality of life. They offer students of all ages opportunities to embark on adventures that stimulate the intellect and imagination.

Policy statement on Mathematics and Science Education, Massachusetts Board of Education, 1992

Envision a society whose members understand and appreciate the value of science and mathematics—a society where the people have the knowledge, positive attitude and technological skills to contribute new ideas in a complex world. Envision schools where all learners—not just those who aspire to be scientists or mathematicians—discover the power of mathematics, science and technology and develop the ability to reason scientifically and mathematically as they investigate and solve complex problems using the tools they need.

In the brief scenarios below, compare the work students are doing in the classroom in Quincy with the work a research team is doing in a business environment.

A Classroom

A fourth grade class in Quincy is trying to determine optimal growing conditions for bean plants. In small groups, students brainstorm and try out different theories, measuring and testing variances in amount of water, sunlight, and soil composition. Different group members organize the data collected into charts and graphs; others record the group's findings in a journal. The groups present their predictions and results to the class.

A Business Workplace

A research team at a photography company is trying to find a more environmentally-friendly package for their film. The team brainstorms ideas, uses on-line computer services to research information on various materials, develops prototypes using the different materials, and conducts focus tests with consumers. The team calculates expected development costs and projects the impact on sales. The team then presents its findings and recommendations to upper management of the company.

In the above example, students in the fourth grade class are behaving in ways that are similar to those of the research team—they are *doing* mathematics and science. The students are investigating a question that requires them to experiment, propose and test various hypotheses, and record, interpret, and present their findings to others.

In Massachusetts, as elsewhere across our nation, mathematical and scientific literacy is becoming increasingly important both socially and economically. As more jobs are transformed by technological change (Zuboff, 1989), low-skill jobs are increasingly becoming low-wage jobs. The economic benefits of education have increased substantially over the last decade (Murnane and Levy, 1993). Employees with college degrees have always earned more (on average) than those whose highest qualification is a high school diploma, but current economic research is showing an increasing gap between the earnings of college graduates and those with only high school education. Additional work by these researchers also suggests that among students who leave school with only a high school diploma, those with higher scores on mathematics tests tend to earn more during their first five years of employment than those whose scores are mediocre (Murnane, Levy and Willett, 1993). These findings suggest students' future economic well being is not only affected by whether they stay on or drop out, but also by what they learn and how well they learn while they are at school. In light of these findings, it is critical that equal access to quality mathematics and science courses be available to all students.

While the economic incentives for scientific and mathematical literacy are compelling, they are not the only force driving the public schools toward a greater emphasis on science and mathematics learning. More than ever before, the citizens of Massachusetts are being asked to make difficult choices between economic development, customary life-style choices, and emerging environmental issues. As the Massachusetts Common Core notes in its preamble," Not so long ago, most Americans did not worry about their environment. Now, with the global population explosion, worldwide industrialization, increased use of natural resources and the degradation of rain forests and agricultural land, students need to develop skills to analyze the environmental issues that face them today and that will challenge them tomorrow."

Consumers are bombarded by claims and counterclaims about energy efficiency, alternative forms of energy, waste management,

recycling, and other issues which challenge aspects of the life-styles we take for granted. Being able to think scientifically and use scientific information responsibly is an important part of being a citizen in our increasingly complex world.

The social and economic implications of access, or lack thereof, to quality mathematics and science education constitute a compelling case for providing comprehensive and effective programs in the Commonwealth. Indeed, the writers of these Frameworks highly recommend that all Massachusetts students be enrolled in a science class and a mathematics class, every year, from Kindergarten through Grade 12. This and related recommendations regarding the teaching of all sciences every year in Grades K-10 and the selection of alternative pathways in Grades 11 and 12 are discussed in "Science and Technology," Chapter 3.

Curriculum Frameworks: Tools for School Improvement

As expressed in the Massachusetts Education Reform Act of 1993, a paramount goal of the Commonwealth is "to provide a public education system of sufficient quality to extend to all children the opportunity to reach their full potential and to lead lives as participants in the political and social life of the Commonwealth and as contributors to its economy" (Section 27, p. 10). In support of this goal, the act authorizes the development of curriculum frameworks.

A curriculum framework is a guide for districts, schools, institutions of higher education, and teachers to use in planning and evaluating programs. It identifies the important content for a subject area and also addresses how this content should be taught.

Frameworks Are:	Frameworks Are Not:
A guide to assist districts and schools in designing their mathematics and science curriculums	A detailed syllabus or curriculum
Suggestions for improving current mathematics or science education	A mandate for mathematics or science education
Suggestions for assessment	Items to "teach to the test"
A guide for selecting materials	A directive for uniform texts
A description of important mathematics and science that students need to know and be able to do	A list of expected outcomes
A presentation of critical issues about what we teach and how we teach it.	Solutions to the problems.

The Massachusetts Curriculum Frameworks in Mathematics and in Science & Technology do not contain the specific detailed curriculum that is actually taught in each school. Rather, they set out broad curricular standards in multi-year blocks, communicating the core understandings that are essential for students to learn in mathematics and science. Each district, each school, and each classroom teacher will need to determine how these curriculum standards can be attained for their students.

Mathematics and Science Curriculum Frameworks

The Massachusetts Frameworks have their roots in the national movement for curriculum reform as well as in national and international academic standards. The Mathematics Frameworks are based upon documents developed by the National Council of Teachers of Mathematics (NCTM): *Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics,* and the working draft of *Assessment Standards for School Mathematics.* The Science Framework draws from efforts by the National Research Council (NRC), *National Science Education Standards*; the American Association for the Advancement of Science (AAAS), *Benchmarks for Science Literacy*; and the National Science Teachers Association (NSTA, *Scope, Sequence, and Coordination of Secondary School Science).*

PALMS Principles for Learning and Teaching

The Massachusetts Frameworks for Mathematics and Science and Technology are based upon a set of principles—a vision of learning and teaching. This vision was developed by PALMS Mathematics and Science Specialists (classroom teachers, university professors, museum educators, and other mathematics and science educators) during a professional development institute in Fall 1992.

- *All children can learn.*
- *Mathematics and science instruction should be based on the know edge that learners construct their own meanings.*
- *Mathematics and science instruction should emphasize the quality of understanding rather than the quantity of information presented.*
- *Assessment should be used as a tool to improve instruction and enhance student learning.*
- *Learners need the social and organizational skills developed by working in groups. Working in groups helps learners make sense of science and mathematics through communication. Learners benefit from social, organizational, self-evaluative, and small group endeavors.*

- *Learners learn best in an environment which acknowledges, respects, and accommodates each learner's backgrounds, learning style, and gender.*
- *Learning is a lifelong process that begins and continues in the home and extends to school and community settings.*
- *Students learn science and mathematics by engaging in authentic tasks of inquiry, reasoning and problem-solving that reflect real-world scientific and mathematical practice.*
- *Hands-on experiences deepen understanding of abstract concepts by encouraging the practice of process skills and communication and allowing for reflective thinking.*

Chapter 1: Critical Issues of the Frameworks

Strengthening our current practices in school is a multifaceted and complex undertaking. The development of Frameworks for Massachusetts is just the beginning of changing the ways in which mathematics and science are taught and learned in school. There are many other issues that need to be addressed over the next several years as Massachusetts implements the vision described in these Frameworks. Three of the most critical issues are:

- **Equity**
- **External assessment**
- **Professional development of educators**

Equity

> *"We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach. Unless corrected, innumeracy and illiteracy will drive America apart."*
>
> *National Research Council, Everybody Counts, p. 14*

One essential goal that this framework promotes is for all students to be literate in mathematics and science. Success in mathematics and science provides students with access to a wide variety of jobs and enables them to make informed decisions about complex problems that affect society and their personal lives. Scientific and mathematical literacy is possible for all students: students of all races and ethnic backgrounds, male and female students, students from different social classes, students with different levels of proficiency in English, students with and without physical handicaps.

It is clear that there is much work to be done to accomplish the goal of mathematical and scientific literacy for all. By all reports, females, racial and ethnic minority groups and persons with disabilities are trailing far behind mainstream students in mathematics and science. As the following statistics reveal, these groups have typically been under-represented in higher level science and mathematics courses in high schools and universities.

- In 1992, 10% more twelfth-grade Massachusetts boys took science classes than did girls.
- In the same year, 38% of male high school students in Massachusetts, compared with 22% of female students, took physics.
- In a Rhode Island study, 64% of the male students who had taken physics and calculus were planning to major in science or engineering in college, compared to 18.6% of females who had taken the same courses (National Science Foundation, 1990, p. 128).
- Whites and Asians constitute 95% of all Americans receiving graduate and undergraduate degrees in the physical sciences (mathematics, physics and engineering) (National Research Council, 1989, p. 20).
- The average number of African Americans and Hispanics receiving a doctoral degree in the mathematical sciences throughout the USA is less than ten per year (National Research Council, 1989, p. 20).
- Only 17% of the doctoral degrees and 35% of the master's degrees in the mathematical sciences granted in the USA are earned by women, compared to 46 percent of undergraduate degrees in mathematics (National Research Council, 1989, p. 22).

The data suggests a need for a careful scrutiny of students' early experiences with mathematics and science in school. Pre-kindergarten and kindergarten children from different races and social classes possess equivalent informal mathematical ideas and strategies (Ginsberg and Russell, 1981). Even students who have begun to experience difficulty with mathematics in school (disproportionately from a lower social class) showed no evidence of fundamental differences in thinking ability when compared to more successful students (Russell and Ginsburg, 1984). Despite the clear indications of these students' early potential, by the third grade the disparity that persists well beyond twelfth grade begins to appear: African Americans and Hispanics do not perform as well as Whites on national surveys of mathematics achievement (Secada, 1992, p. 628).

A similar pattern emerges in the differences between the performance of male and female students in mathematics and science. Throughout elementary school, girls perform as well as boys on standardized tests, but in secondary school male students begin to consistently outperform females (Maccoby and Jacklin, 1974).

Despite the difficulties that school systems have encountered providing a successful educational experience for African American, Hispanic, female students, and others outside the mainstream, there are clear indications that success with these populations of students

is achievable.

- At one magnet elementary school in Cambridge, 50% of a multi-racial group of students qualified for an accelerated sequence of advanced mathematics courses in high school.
- In Nucalc, a precalculus/calculus course composed overwhelmingly of Black and Hispanic students from public schools in Boston, Cambridge and Chelsea, 85% were successful in precalculus and 74% in calculus.
- In Masspep's Summer High, a program for minority and female students, over 90% of the participants have been enrolled in advanced mathematics and science courses in school following participating in the program from 1990–1993.

The Barriers to Success

There are many factors which can contribute to students' success in learning mathematics and science in school, or can effectively derail them. Some of the factors are related to structures in schools, others are related to how mathematics and science courses are organized and taught, and still others are related to how both teachers and students perceive their roles and how they interact with one another.

Structural Barriers

The structural barriers to disadvantaged students' excelling in mathematics and science are subtle and varied. The most ubiquitous is the maintenance of academic tracks which effectively exclude students from challenging courses over the long term. In a recent paper, the National Governors Association (1993) criticized the faulty assumptions which underlie tracking, and argued that it tends to accelerate the gaps in student learning and create barriers that limit a student's ability ever to catch up.

Students are placed in tracks on the basis of perceived achievement or ability in mathematics and science. Once students are placed in a track, sometimes as early as sixth grade, it is difficult for them to move to a higher track—their course work has placed them further and further behind with little or no means of ever catching up. A growing number of scholars and educational reformers oppose tracking, describing it as a clear barrier to learning (Oakes, 1985; Wheelock, 1992). These Frameworks support this position.

- As practiced, tracking often functions as a form of racial sorting. For example, Oakes (1993) found that students of color were consistently assigned to lower tracks even when they had higher

test scores than white youngsters who were placed in the highest tracks.

- Decisions about placement in one class or another are sometimes based on little information—perhaps on the basis of a standardized test and/or one year's mathematics grade, a method that has not proven reliable for predicting students' subsequent performance in school.
- Over time, students in classes which are labeled "low level" may suffer from low self-esteem in relation to school tasks. They develop poor attitudes toward school, lose their ability to persist in difficult tasks, and, therefore, live up to the expectation that they will not achieve.
- Students (and parents) are often unaware of the impact their choice of mathematics and science courses has for future access to advanced courses and employment opportunities. In many cases minority and female students are not encouraged by guidance counselors to achieve their greatest potential.

Chapter 4 includes some questions for school districts to consider as they address their practices related to tracking.

Content and Teaching of Mathematics and Science Courses

One result of tracking is that different level courses vary considerably. The courses differ in the mathematics and science content, the kinds of problems and work students are expected to do, the instructional strategies used, the qualifications of the teachers, and the access to material resources.

- The most experienced teachers are often assigned to higher level classes and teachers with less skill, experience or knowledge are assigned to lower level classes.
- Students in lower level classes are often given routine, repetitive work. While students in higher tracks learn to think critically, those in lower tracks focus on basic skills and miss the opportunity to study important areas of mathematics and science.
- The focus of many low-track classes has been on remediation based on the assumption that students lack many of the basic skills that are prerequisites for doing higher-order work.
- Teachers who teach classes which are labeled as "lower level" find it difficult to maintain high expectations for their students. Lower expectations, predictably, lead to lower achievement.

A recent study funded by the National Science Foundation that analyzed the six standardized tests most widely used in the U.S. found that at least 95 percent of the mathematics questions and 75 percent of the science questions were low-level (such as recall and

routine application) rather than high-level (such as reasoning and applying concepts). This directly impacts the mathematics and science content that is presented in high-minority classrooms, since the same study reports that teachers with high percentages of minority students spent more time on topics that were included on standardized tests than did teachers of low-minority classrooms (Center for the Study of Testing, Evaluation, and Educational Policy, 1992).

Teachers' and Students' Assumptions and Interactions

How each individual perceives the world is influenced by the person's race, gender, culture, and social class. This is true for teachers and other adults as well as for students. Teachers' and school administrators' assumptions about their students' academic potential, their intentions, motivations and behaviors often lead to interactions which do not support the development of students outside of the mainstream. Every group that does not fit the mainstream experiences bias and some form of inequality. These biases can be very subtle. Educators are often unaware of their own biases or how these biases affect individuals' behavior.

Gender: One example of differentiated teacher/students interactions comes from research on gender. Males tend to receive more attention from teachers, more feedback, and longer wait time for considering higher-level, as opposed to lower-level, cognitive questions. In general, students' interest in and enthusiasm for mathematics and science declines the longer they are in school, with losses far greater for girls than for boys (AAUW, 1992). Gender differences in students' confidence in their ability to do well in mathematics and science also increase with age. As boys get older, those who do not like mathematics are likely to attribute this feeling to the subject itself; girls, instead, interpret their problems with mathematics as personal failures (Fennema, 1977). While most researchers concede that teachers may be reacting to behaviors students bring to the classroom, they also conclude that these reactions reinforce stereotypical behaviors.

Race and Ethnicity: Few educators are conscious of how they are influenced by racial and ethnic prejudices. However many students, especially inner city youth, still experience the pervasive and subtle effects of these prejudices. For example, educators may be unfamiliar with the culture and daily issues that African American, Hispanic and students of other racial or ethnic minorities face and may have low academic expectations for them. In this situation it is under-

standable to blame poverty, family disintegration, and violence for students' difficulties in school. But do these societal problems explain why a talented black teenager in a suburban school was assigned to a low-level chemistry class, or why an obviously enthusiastic hard-working teacher consistently picks white students to answer her questions, avoiding the outstretched hands of the two black students directly in front of her? Erroneous assumptions about students may impact them negatively, a sobering thought when one considers the lost potential for these students.

Jamie Escalante's work with inner city Hispanic students is one example of an effective approach. He has demonstrated that these students can be successful in the study of calculus; a similar conclusion may be drawn from Northeastern University's Nucalc program for African American and Hispanic students. Marva Collins of Chicago has proven that African American children from poor disadvantaged families can understand the difficult material in the classics, yet the myth survives that the vast majority of urban poor, and in particular African Americans, are slow learners, who have very little chance of success except in rare cases.

Language Minority Students

Language minority students constitute a significant and growing percentage of students enrolled in Massachusetts schools. In 1990, one student in ten attending Massachusetts public schools had a first language other than English. Students entering schools speak over 70 different languages (FY 1989 Massachusetts Annual School Report). A majority of language minority students are placed in all-English classes before they are sufficiently proficient in the language. Many are able to converse in English and appear to understand what the teacher and classmates say. Yet they often lag behind native English speakers in academic work, particularly on tasks that require reading, writing and comprehension. This has a significant impact on how these students perform in mathematics and science classes, particularly as the curriculum focuses more and more on in-depth investigations with students communicating their thinking orally and in writing.

One reason language minority students have difficulty in school is that while conversational language proficiency takes about one to two years to develop, the development of academic language proficiency can take anywhere from four to eight years. Another factor is the amount of schooling students have received in their native

language. Numerous studies show that students who have had formal schooling in their native country, and are literate in their primary language, tend to do better than students who have had all their schooling in their second language—English. (Massachusetts Department of Education, 1990)

Furthermore, many immigrants come from cultures and societies whose languages, religious philosophies, economic structures, and patterns of behavior bear little or no apparent resemblance to what is common in the United States. For example, a new student may come from a society in which the societal status distance between teacher and student may be very different.

A Shared Challenge

The issue of assuring equity is a difficult one. What is the responsibility of the school system and individual teachers? What is the responsibility of the community and government? We can easily say that there must be equal opportunity for people of all races, socioeconomic backgrounds, gender, ethnicities, religions, and intellectual or physical capacity. But what does that mean? How is it accomplished? How do you know when it has been achieved? It is impossible to provide definitive solutions to such a complex issue. However, several ideas and suggestions for how teachers, schools, and districts might begin to address this question are provided in Chapter 2, "Learning, Teaching, and Assessing Mathematics and Science," and in Chapter 4, "Implementing the Framework."

External Assessment

External assessment refers to student tests that are not developed by the classroom teacher as part of the instructional program—they are generally developed by textbook and test publishing companies or a state. External tests provide information about certain aspects of student achievement. This information frequently has been used to monitor the progress of a student or group of students, and educational programs in classrooms, schools, and/or states.

It is well documented that external tests have a significant impact on what happens in classrooms. Teachers feel a need to ensure that their students do well on these tests, and often modify their teaching practices to match what is assessed. Therefore, it is imperative that

the kinds of external assessments used be aligned with the kinds of curriculum desired. Very few of the current external tests match the vision of this Framework.

> *"As long as the mathematics content of standardized tests differs from the mathematics curriculum called for in the NCTM Standards, teachers faced with mandated testing will find themselves in a difficult position. For example, even teachers who recognize the benefits of calculators often justify their reluctance to use them in their mathematics classes by arguing that students are not allowed to have them while taking standardized texts".*
>
> *Hancock, L. and Kilpatrick, J., 1993, p. 165.*

As described in *Measuring What Counts* (Mathematical Sciences Education Board, 1993), there are three fundamental principles that should form the foundation of all assessment that supports effective education.

- **The Content Principle:** Assessment should reflect the mathematics and science that is most important for students to learn.
- **The Learning Principle:** Assessment should enhance mathematics and science learning and support good instructional practice.
- **The Equity Principle:** Assessment should support every student's opportunity to learn important mathematics and science.

The following guidelines are suggested for states and districts to use when designing, developing, or selecting external assessments.

The Content Principle: Assessment should reflect the mathematics and science that is most important for students to learn.

The most important ideas of mathematics and science are described in this framework and the NCTM *Curriculum and Evaluation Standards;* the NRC *National Science Education Standards,* and the AAAS, *Benchmarks for Science Literacy, Project 2061.* These documents broaden the definition of content knowledge to include scientific and mathematical processes. Emphasis is also placed on making connections within the discipline as well as across different disciplines.

Rather than focusing only on low-level skills and techniques, external assessments should include opportunities for students to engage in longer, in-depth problems and investigations where they can demonstrate what they know, understand, and are able to do. Generally, these tasks are open-ended, with many approaches possible, and they frequently have more than one correct solution. These tasks may ask students to use what they have learned in a new

context or situation; others may require students to interpret data or results in a variety of ways. Students may be asked to write explanations of their thinking while doing the task and to justify results.

How tasks are scored is very important. "The interpretation of responses to assessment tasks must allow for students' use of alternative approaches and for recognition of the important mathematics that has been used. In particular, when students are communicating the results of their work, the response that expounds on many themes but never grasps the heart of the problem should not be valued over the elegant but terse solutions" (NCTM, 1993, p. 31).

The Learning Principle: Assessment should enhance mathematics and science learning and support good instructional practice.

If students are expected to work with others in class, but they are always assessed individually, they will learn implicitly what is valued. If students are expected to use tools such as manipulatives and calculators in class, but they are not permitted to use these tools on assessments, they will learn what is valued. If students are expected to explain their reasoning in writing in class, but they are only assessed on multiple choice tests, they will learn what is valued.

As much as possible, external assessment should be consistent with instructional practices. Tasks on external assessments should be similar to the kinds of investigations students do in class. Students should have opportunities to work on complex problems, at times with others and at other times alone. (Students, for example, may do an investigation with others, but individually write up the results and conclusions.) They should have the tools they need.

The Equity Principle: Assessment should support every student's opportunity to learn important mathematics and science.

> *To meet the equity principle, tasks must be designed to give children a sense of accomplishment, to challenge the upper reaches of each child's mathematical [scientific] understanding, and to provide a window on each student's mathematical [scientific] thinking. Just as good instruction accommodates differences in the ways learners construct knowledge, good assessment accommodates differences in the ways that students think about mathematics [science] and display mathematical [scientific] understanding....(T)he equity principle implies that assessments must be sufficiently flexible to allow all students to show what they know and can do.*
>
> *Mathematical Sciences Education Board, 1993, p. 92*

"Some ways of accommodating differences among learners include permitting multiple entry and exit points in an assessment and allowing students to respond in ways that reflect different levels of mathematical [scientific] knowledge or sophistication." (Mathematical Sciences Education Board, 1993, p. 93) Some students may need to have alternative ways of accessing the test and/or alternative ways of responding. For example, students who have reading or visual impairments might have the test read to them; students who are limited-English speaking, might respond in their native language.

Professional Development

> *A high quality public education system depends upon educators who engage in continuous professional growth and development. To that end, the Board and the Department of Education are committed to the creation and ongoing support of schools and other educational settings as "communities of learners" which encompass both students and the adults involved in their education.*
>
> *Massachusetts Department of Education, 1994, Statewide Plan for Professional Development, p. 1*

Implementing this Framework requires much more than its adoption and dissemination by the State Board of Education and local school districts. Implementing the Framework's recommendations requires extensive and ongoing programs that are considered essential. Professional development programs need to support teachers and other educators as they examine their current practices, learn and try out new strategies and approaches, and reflect upon their efforts and the results they see with their students. This process of professional development is recursive; as educators, they will try out new approaches and reflect upon them and see other issues and approaches to examine. Educators are also learners, continuing to deepen their understanding of subject matter and their understanding of students and how they learn.

Schools and districts need to take considerable time in designing and planning professional development programs. High quality professional development is characterized by the following:

- Time and resources (to plan, engage in, and assess professional development, including follow-up and support activities.
- Collegiality and collaboration.
- Experimentation and appropriate risk-taking.
- Opportunities for reflection.

- Incorporation of available knowledge bases on teaching and learning (in both general strategies and specific content areas).
- Participant involvement in goal-setting, implementation, evaluation, and decision-making.
- Leadership opportunities, including shared leadership.
- Sustained administrative support.
- Appropriate incentives and rewards.
- Designs built on principles of adult learning/development and the change process.

Ibid, p. 3.

These characteristics of professional development are discussed in more detail in Chapter 4, "Implementing the Framework."

Organization of the Framework

This chapter highlights some of the most critical issues associated with making changes in mathematics and science education in the Commonwealth. Chapter 2 discusses principles of learning, and how effective teaching and assessment practices related to how students learn mathematics and science. Chapter 3 presents the important mathematical or science content that students should understand and be able to do. The chapter includes examples of activities and classroom practices that illustrate how students might engage with the content. Chapter 4 describes some of the factors, such as partnerships and professional development, that are related to changing the ways in which mathematics and science are taught.

Chapter 2: Learning, Teaching, and Assessing Mathematics and Science

The content standards set forth in Chapter 3 of this framework present a vision of what every child educated in the Massachusetts public schools will know and be able to do. All students are expected to achieve the standards, and all districts, schools, teachers, and community members are expected to provide the support, resources, and time that each child needs to do his or her best.

The time and pathways that students and teachers use to reach the goals set forth in the standards will vary according to the needs of each classroom. However, all pathways share an underlying set of principles based on knowledge about the process of learning and the strategies of teaching and assessment.

This chapter presents a set of key principles (drawn largely from the PALMS principles) to describe the characteristics of learners and the nature of teaching and assessment. These principles are supported by findings from research in cognitive science, and mathematics and science learning research. Implemented as a whole, these principles create a learning environment and community in which all students can attain mathematical and scientific understanding. While the principles highlight and discuss separate ideas, the components of learning cannot be isolated. Learning is a complex process that takes place in a complex environment, which arises out of an interplay among principles.

Principles for Learning, Teaching and Assessment

- *All children can learn.*
- *Learners construct their own meanings.*
- *Learning is a life-long process that begins and continues in the home and extends to school and community settings.*
- *Learners learn best in an environment which acknowledges, respects, and accommodates each learner's background, learning style, and gender.*
- *Mathematics and science instruction should emphasize the quality of understanding rather than the quantity of information presented.*
- *Students learn science and mathematics by engaging in authentic tasks of inquiry, reasoning and problem-solving that reflect real-world scientific and mathematical practice.*
- *Hands-on experiences deepen understanding of abstract concepts by encouraging the practice of process skills and communication and allowing for reflective thinking.*
- *Learners need the social and organizational skills developed by working in groups. Working in groups helps learners make sense of science and mathematics through communication. Learners benefit from social, organizational, self-evaluative, and small-group settings.*
- *Technology should be used as a tool for learning mathematics and science.*
- *Mathematics and science instruction should emphasize connections within and across disciplines.*
- *Assessment should be used as a tool to improve instruction and enhance student learning.*

...[S]cience teachers should encourage students to raise questions about the material being studied, help them learn to frame their questions clearly enough to begin to search for answers, suggest to them productive ways for finding answers, and reward those who raise and then pursue unusual but relevant questions. In the science classroom, wondering should be as highly valued as knowing.

Rutherford & Ahlgren, 1990, pp. 190–191

...[W]e see classrooms as places where interesting problems are regularly explored using important mathematical ideas. Our premise is that what a student learns depends to a great degree on how he or she has learned it. For example, one could expect to see students recording measurements of real objects, collecting information and describing their properties using statistics, and exploring the properties of a function by examining its graph. This vision sees students studying much of the same mathematics curriculum currently taught but with quite a different emphasis; it also sees some mathematics being taught that has received little emphasis in schools.

NCTM, 1989, p. 5

The classroom of tomorrow

Classrooms that reflect the principles of this framework can be found in many schools across the state. In these classrooms, students are engaged in finding solutions to complex, real-world problems. They use the content and strategies of mathematics and science to define the problem, identify possible solutions, collect and analyze data, communicate their thoughts and ideas with others, and act on their conclusions. The students work both independently and collaboratively, using hands-on materials and technology as tools to assist them in their work. They show curiosity, initiative, perseverance, self-discipline, and ingenuity. In the inquiry-based classroom, students have many opportunities and are supported to take responsibility for their learning.

The teacher plays many different roles in a problem-solving oriented classroom: questioner, facilitator, decision maker, reasoner, resource provider. The teacher observes students, listens to what they say, assesses what they understand, provides guidance and feedback, and uses multiple strategies and methods based on the needs of the students. The teacher makes decisions, selecting rich problems and methods of instruction, and providing resources that will engage learners of different styles, abilities, backgrounds. The teacher facilitates, encouraging students to formulate their own questions, and models the spirit of inquiry through asking questions.

The teacher encourages multiple and alternative ideas by asking "What other ideas do you have?" or "What's another way of doing this?" The teacher encourages students to explain their thinking and elaborate by saying, "You need to tell me more, I don't yet understand." The teacher encourages children to reflect by asking "What do you know now? What would you do differently? What will you do next? Where can you get more information?" The teacher imparts knowledge and demonstrates procedures needed to aid the students in understanding and solving problems.

Both teacher and students have high expectations for the level of student work and for their products. All members of the classroom communicate these expectations clearly and frequently, as they reflect on the quality of the work being done and the students' understandings. Both teacher and students observe and assess students' progress and understandings informally. When students are ready, they more formally present their understandings for assessment as written solutions, group and class presentations, or public exhibitions.

All children can learn.

> *Equity in education means creating an environment in which all students can prepare themselves for the challenge of leading a productive life. Schools, parents and students need to recognize that each and every student has the capacity to excel. The reciprocal advantages of students' different backgrounds, perspectives, learning styles and abilities can be validated by the school to benefit learning. Educators can encourage the development of bonds between schools and communities, between teachers and students, and between student and student.*
>
> *Schools can foster equity by creating initiatives and programs which take into consideration race, gender, national origin, physical condition or attributes, age, class and/or socioeconomic status to ensure full participation in school life.*
>
> *Instruction and Curriculum Services,*
> *Massachusetts Department of Education, September 1993*

All children have the capacity and the initial motivation to learn, to reason, and to seek out new experiences.

Children who come from a different culture, or from an economically disadvantaged subgroup within the American culture, often do not have access to the same opportunities and experiences as their schoolmates, or may have different views or values about those experiences. The same is true for some girls who are not encouraged to engage in mathematics, science, or technology. The family, religious, and social culture in which children are raised also influence what things they think are important to learn, and at what age.

Learners' beliefs about themselves, about learning, and about the particular subject matter will influence whether they will be motivated to learn, open to new ideas, or even be familiar with ideas that other children already have mastered. More importantly, children develop their beliefs about themselves from the community they live in and the way other people interact with them. Research has consistently shown that the opinions and expectations of a teacher or other adults and classmates will have a direct or indirect impact on the motivation of a child and the opportunities made available to that child (Rosenthal & Jacobson, 1968). Many of these opinions and expectations are quite subtle, and not easily noticed or corrected.

While progress has been made in ensuring that equitable programs provide resources to children who need them, too many children in Massachusetts are still not reaching their potential. It is the responsibility of every person in the state to examine his or her own beliefs, and every community to examine its educational pro-

grams to insure that children of all walks of life are fairly represented in all programs, and have equitable access to all resources. Some children may ultimately progress further in their learning than others, and learning may take different amounts of time for different learners, but each child *can* learn.

Learners construct their own meanings.

Children learn through active experience, by playing and working by themselves, with other children, and adults. Each child's interpretations of a new situation are shaped by his or her past experiences with concrete objects and real-world events, and are influenced by culture and social opportunities.

Every individual creates meaning by synthesizing new experiences with his or her prior understandings of the world. The learner interprets new information in light of what he or she already knows or perceives to be true. When new information is consistent with the knowledge and perceptions already held by the learner, he or she can accommodate this information. If, on the other hand, the new information does not initially "fit" with the learner's previous understandings, he or she must reconcile the discrepancy.

We either interpret what we see to conform to our present set of rules for explaining and ordering our world, or we generate a new set of rules that better accounts for what we perceive to be occurring. Either way, our perceptions and rules are constantly engaged in grand dance that shapes our understandings.

Brooks and Brooks, 1993, p.4

Misconceptions, contradictions in understanding, or confusions are common; and they are a critical part of the ongoing process of "coming to know." A prime opportunity for learning occurs when a learner recognizes that there are discrepancies between what he or she currently understands and new information that he or she encounters. The learner struggles to reshape his or her understandings in order to incorporate the new information into his or her conceptual framework. The result is a refinement and reorganization of knowledge—a new way of thinking and a deeper understanding. It is through engagement with ample rich experiences that challenge a learner's existing ideas that a learner gradually modifies or replaces his or her understandings with new, more sophisticated concepts.

Learning happens not only through experience, but also through reflecting on that experience. Learners of all ages gain new understandings as they think about their ideas and describe and share their ideas with others. Through active communication and reflection on their experiences, people develop broader and deeper understandings, and also become aware of how they learn.

Learning is a lifelong process that begins and continues in the home and extends to school and community settings.

The need to make sense of the world begins before school, and continues outside and beyond formal schooling. The tenets of these Frameworks apply not only to students in Grades K–12, but to the needs of pre-kindergarten and adult learners as well. Each of these populations has several distinct characteristics.

Mathematics and science in early childhood

Young children begin to form ideas about mathematics and science at a very early age as part of the natural process of exploring their world. For example, building with blocks gives young children an opportunity to develop an understanding of shape, size, position and symmetry; by collecting leaves and shells and observing animals, they learn about patterns and classification. These types of explorations are necessary precursors to a complete understanding of mathematics and science.

The National Association for the Education of Young Children (NAEYC) makes the following recommendation about the curriculum of early childhood programs:

> *Learnings about math, science, social studies, health, and other content areas are all integrated through meaningful activities such as those when children build with blocks; measure sand, water, or ingredients for cooking; observe changes in the environment; work with wood and tools; sort objects for a purpose; explore animals, plants, water, wheels and gears; sing and listen to music from various cultures; and draw, paint, and work with clay.*
>
> *NAEYC, p. 56*

Preschool activities give children opportunities to solve problems and discover information for themselves in an environment where they can explore freely and safely. Young children need repeated experiences exploring materials, time to talk about their experiences, and freedom to make mistakes and learn from each other.

Early childhood vignette

A group of 3- and 4-year olds have been collecting interesting items from their playground. When they come into the classroom, the teacher puts three trays on a table. She helps them sort the items they have found according to their ideas: "soft" things (leaves and grass), "hard" things (stones), and "junk" (bottle caps, cans, bits of paper). Tracey "counts" the stones on the tray: "One, four, seven, eight, thirteen, twenty-nine..."

Later in the morning, these same children have been asked to "vote" on which story to read at group time: Red Riding Hood or Snow White, by dropping a Unifix cube into a dish in front of each book displayed on a table. At group time the teacher has children count the cubes and assemble them into stacks. The children then talk about the stacks—size, number—and collectively decide which book should be read to the group. The teacher then records the information on a chart that keeps track of the things children vote on. The numbers become a vital part of the life of the classroom.

Adult learners in mathematics and science

Each year, over 3 million adults in the United States enroll in adult basic education courses. The Adult Basic Education Math Standards Project (1994) identifies several characteristics of adult learners that impact how they will learn math and what math they will need to learn. Most of these characteristics apply to adult learners of science as well. Adult learners seek further education to meet a specific goal: to do better in their job, to help their children, to improve themselves. These adults all seek improvement in their skills of mathematics and science that will apply to their lives in the present. At the same time, each individual has unique functional abilities and life experiences, including the daily pressures of life all adults carry.

Instructors of adults can use the principles set forth in these Frameworks to support the learning of adults in the same way they support children: by offering hands-on problems based on real life situations, and providing a range of manipulatives and technologies. The adult learner should be considered an active participant in defining personal learning objectives and deciding measures of success. Instruction should include opportunities to question, discuss, and write about ideas, and to experience success, and must be free of cultural bias.

An ABE instructor shares her experience:

> *The learners' inquiries into the formation of graphs lead to more work...they were instructed to bring in graphs of interest to them....There were bar graphs on everything from consumption of candy bars per person per year, to circle and bar graphs on the concentration of wealth in the United States...and most important, [on] how the less educated were suffering more. This last one had a great impact upon my learners, for they noted the importance of a high-school diploma and its connection to a better income.*
>
> *The ABE Math Standards Project, 1994, pp. 27–28*

Learners learn best in an environment which acknowledges, respects, and accommodates each learner's backgrounds, learning style, and gender.

All learners benefit from encountering important ideas in a variety of contexts and instructional strategies. While the stages of child development are fairly predictable across a range of people, each individual child will develop at his or her own pace. Not all learners at a given age will share the same knowledge, skills, life experiences, cultural outlook, access to resources, or learning styles.

Children have different learning styles that influence what kinds of tasks and forms of information they prefer. For example, a child may not be comfortable with writing but may excel at drawing or building a model that shows his or her understanding of an idea.

One way to help all learners in Massachusetts schools to reach their full potential is to treat the diversity in communities and classrooms as a gift. As discussed above, children learn best when they can connect their real-life experiences to classroom learning, and when they can experience the same concept or idea in multiple contexts. It therefore makes sense to embed problems or investigations and draw resources from the various cultures and backgrounds of different students, to encourage students to work collaboratively in groups of mixed background and ability, and to present as role models women, minorities, and people with disabilities who have made important contributions as scientists and mathematicians.

At the same time, the presence of diverse learners in Massachusetts classrooms presents an opportunity for all students and teachers to learn more about the rest of the world and appreciate the talents and culture of each individual. Since different cultures sometimes use alternative mathematical strategies or perceive the relationships of objects and events in the world in ways other than the

mainstream culture, their strategies and understandings can enrich the understanding of all students. For example, Cambodian children learn a different algorithm for division. If given a chance to explain their method to the rest of the class, then there is an opportunity for all not only to increase their understanding of the concept of division, but to value the varied approaches to mathematics.

Readers: We need parallel example for science.

While a discussion of specific strategies to accommodate the needs of different groups of learners is beyond the scope of this Framework, it is important to state that in general, engagement in real-world problems using a variety of methods and contexts in heterogeneous groups is an instructional model that has been shown to support virtually all learners in achieving greater depth of understanding and literacy in mathematics and science. There are particular populations of learners in Massachusetts, however, that are receiving increased attention by mathematics and science educators because of recent research which indicates that many members of these populations fail to participate or continue to participate below their capabilities.

Language minority learners in mathematics and science

Children who arrive in our schools with a different cultural background and language may fall behind in subjects while trying to master English. More recently, educators have begun working with these children to study mathematics and science in their native language, while also teaching them English. The ability to learn subjects through English is "best developed through initial subject matter instruction in the native language [of the learner] with a gradual transition into English instruction" (Massachusetts Department of Education, 1990). Giving language minority children opportunities to learn and be assessed in their native language will give teachers and other educators a much clearer view of these children's understandings, which will then increase their opportunity for participation in the regular classroom.

Cheche Konnen: Scientific Sense-Making in Education

Cheche Konnen is the name that was given to an innovative urban science and mathematics program to demonstrates the power of a sense-making approach for language minority students. The project began in 1988 in Cambridge, at the Graham and Parks Alternative Public School (K-8) in a Haitian bilingual 7th an 8th grade class, and a multilingual, basic skills high school class. It has since expanded to Haitian Creole and two-way Spanish-English bilingual classes in grades 4-8.

In Haitian Creole, Cheche Konnen means search for knowledge. In Cheche Konnen what students think is at the center of their activity. They explore their own questions, design studies, collect, analyze and interpret data, build and argue theories, and evaluate evidence. Investigations are interdisciplinary; science, mathematics and language (talk, reading and writing) are linked, and recognized as essential tools of scientific inquiry. Through this approach, language minority students learn to think, talk, and act scientifically. Language mediates their learning in powerful ways.

In their Water Taste Test investigation, for example, students used both Haitian Creole and English to construct scientific meaning and communicate their understanding. Using Haitian Creole, they designed their studies, interpreted data, and argued theories. Using English they collected data from their mainstream peers, read standards for interpreting test results, and reported their findings.

The students were often isolated from the mainstream school community. However, as they conducted their investigations mainstream peers began to see them as doing something important and as experts. Their teachers also took notice of them. Most important, the students developed confidence in their abilities and took a more critical stance to knowledge.

Their teacher, Josiane Hudicourt Barnes, characterized the changes she observed in this way: "I think that the kids' way of seeing the world, the way they think in general, has changed because they feel more comfortable learning on their own, investigating questions, thinking about questions, making them clearer, and finding out answers whether from books or from experimentation. And most of all, I feel that they have made a step towards being critical about what people say to them...They are learning to find out for themselves and not listen to everything that they hear."

Adapted from Massachusetts Department of Education, Common Competencies: The promise of linguistic/cultural diversity, 1990, pp. 30–31.

Encouraging females and minorities in mathematics and science

Current research about higher education and the workplace show that minorities and women are consistently under-represented in mathematics and science careers. At the same time, mathematical and scientific literacy has become a necessity for higher-level employment and full participation as a citizen. While virtually all students begin the elementary grades on an equal footing, by high school both girls' and minorities' participation and performance has fallen behind that of white males. The prevailing view in current culture is that the mathematician or scientist is a white male.

Society, parents, teachers, and males and females all must reexamine their beliefs and expectations about who should participate in mathematics and science. Research publications on reforms in education such as *Everybody Counts* (National Research Council, 1989) raise awareness of the role math and science will play in future careers. In general, interventions should focus on increasing all students' confidence, perception of ability, and their persistence on complex problems. Providing role models of women and minorities in mathematics and science can also be helpful. Many females' and minorities' achievements in mathematics appear to improve when they participate in cooperative activities and investigations (Petersen & Fennema, 1985; Secada, 1992). Professional development activities that help teachers reflect on their behavior towards individuals in the classroom will help improve understanding of the problem and provide support for teachers to change their practice (Mark, 1992).

Mathematics and science instruction should emphasize the quality of understanding rather than the quantity of information presented.

Understanding—rather than simply memorizing terms, procedures, and rules—should be the primary goal of mathematics and science instruction. Every teacher is familiar with the child who can give the appearance of having learned something by repeating adult statements or memorizing vocabulary, only to have the child's "knowledge" fall apart under questioning or in a new experience. Understanding involves reasoning, making connections among facts and among concepts, and applying knowledge and processes in new situations.

To help students develop understanding of mathematical or scientific concepts, teachers embed concepts in problems or situations that call for students to think and reason. Understanding is

deepened and made more complete only when teachers provide students opportunities to explore concepts in depth and via multiple approaches. Further, linking of concepts is essential so that learners can gain an understanding of the "big ideas" of science and mathematics. Curriculum and instruction should focus on making explicit the connections among concepts—within a particular unit, across units in a given grade level, and across grade levels.

In the classroom that promotes quality of understanding, the teacher gives students responsibility for formulating and solving problems, for reflecting on their findings, and for communicating about important scientific and mathematical ideas. Students develop knowledge by "doing" mathematics and science—in ways that reflect the kinds of thought and action that are typical of these fields.

Students learn science and mathematics by engaging in authentic tasks of inquiry, reasoning and problem-solving that reflect real-world scientific and mathematical practice.

Both mathematics and science are fields of knowledge that are characterized as much by their processes as by their content. Both mathematical and scientific literacy require the ability to:

- Identify and describe problems
- Choose the appropriate mathematical operations or scientific procedures to investigate problems
- Identify the theories and concepts that are relevant or irrelevant to a problem
- Work by oneself and with others to solve problems.

What students learn has a great deal to do with *how* they learn it (NCTM *Standards*, 1989; Rutherford & Ahlgren, 1990.) The best way to become a problem solver is to engage in solving problems, especially those that mirror the complexity and importance of real-world situations. Students need opportunities to grapple with problems, discuss alternatives, share results, have those results challenged by others, and reflect on their experiences. Students need to work on problems that may take hours or weeks to solve, on problems that are open-ended, and on problems that have multiple solutions.

Educators can support students by creating schools and classrooms where asking questions, raising ideas, observing, measuring, collecting data, working with objects, materials, and numbers are a natural part of daily instruction. Providing students with problems that are connected to their lives and have social impact motivates all students, including minorities and girls, and encourages all students

to view science and mathematics as relevant and connected to their lives. Teachers who develop the habit of listening to the voices of their students and welcome student contributions from their prior experiences outside of the classroom are more likely to encourage good questions from their students. When students suggest questions, they feel more ownership of an investigation, and teachers can get a clearer idea of students' understandings.

Inquiry-based classrooms may not be neat and quiet, but they can be productive, well-managed, and well-provisioned. Structures and schedules within some schools are being changed to accommodate the longer periods of time and flexibility of staffing that are necessary to support an inquiry-based learning environment. While prescriptions about school structuring are outside the recommendations of this Framework, it is important to note that in general, teachers and students need longer periods of concentrated time and freedom from frequent interruptions during the school day in order to engage deeply in ongoing investigations. Also, many teachers and students have had little opportunity to practice inquiry in mathematics and science. Students will need consistent support and opportunities to develop the habits of mind and skills necessary to become responsible for their learning. Teachers will need extended time and flexibility to gather resources, plan, and support each other. See Chapter 4, "Implementing the Frameworks," for more on these matters.

Hands-on experiences deepen understanding of abstract concepts by encouraging the practice of process skills and communication and allowing for reflective thinking.

As described in the section on how learners construct their understandings, knowledge develops in large part from concrete experiences. The more concrete, active experiences learners have, the greater the base they will create for connecting and using concepts and facts. Research in cognitive development suggests that children only gradually develop the ability to represent their concrete experiences more abstractly with thoughts, words, and symbols such as numbers, formulas, pictures, and graphs. This ability to reason through logic and abstraction develops as children work back and forth between their experience and an abstract representation. All learners benefit from continuing to connect concrete, real-world experiences with abstract ideas throughout childhood and even to adulthood.

Therefore it is important that hands-on experiences do not end with an activity, but continue through experiences which promote

discussion of the underlying concepts, presentation, and reflection. Teachers need to provide structures and activities that are minds-on as well as hands-on, integrating process and content, activity and reflection, concrete experiences and abstract representations. Students need opportunities to communicate and share their ideas with others. Hands-on activities provide a rich social context for learning and opportunities to work in groups. The importance of social context and working in groups is described further in the next section.

Learners need the social and organizational skills developed by working in groups. Working in groups helps learners make sense of science and mathematics through communication. Learners benefit from social, organizational, self-evaluative, and small group endeavors.

The social context of learning is critical because it helps students make connections and demonstrate understandings. Learners clarify and justify their ideas when they share their beliefs and have opportunities to know how others are constructing meaning. Learners need opportunities to think reflectively about their work and to discuss their ideas with both peers and people who have more experience.

Communication is an important process of inquiry and reflection. If an idea is not clear when it is communicated, other children can give that important feedback immediately, by asking for more information. They benefit most from feedback and conversation when it is analytical and comes at a time when they are most engaged in the question, as well as after a hands-on experience.

Effective oral and written communication is critical for students to be able to express their ideas clearly, describe their procedures, justify their conclusions, and analyze the work of others. Writing in mathematics helps students describe their ideas while they develop their use of symbols and equations. Writing about data and investigations is the accepted professional mode of presenting and evaluating the validity of new scientific ideas.

Learners benefit from working with others, both peers and adults, in multiple social settings, large and small groups that are heterogeneously mixed (Slavin, 1990). While some tasks are done alone and some people work best by themselves, most people will find themselves needing to work with others in their jobs and their community. The complexity of the problems and amount of information encountered in real-world mathematical or scientific situations

is usually too great for one person to tackle by himself or herself. Several strategies for grouping, such as cooperative grouping (Johnson & Johnson, 1984), have been developed that provide specific structures, roles, and problems for students to develop group work skills.

Technology should be used as a tool for learning mathematics and science.

Technology provides the tools that support both hands-on investigations and reflection and communication about ideas. While technologies in the broader sense (protractors, micrometers, calipers, balances, scales, rulers, calculators) have been used by science and mathematics teachers for years, profound changes in mathematics and science have developed out of the introduction of calculators, computers and related electronic technologies. All students at all levels need to have ample opportunities to use calculators and computers as tools for learning mathematics and science.

The real power of technologies for mathematics and science stems from the use of technology as a tool that the learner and teacher use as a means to an end, where they are in control of both content and process. Tools for learning can be as simple as a pencil and paper or a geoboard, or as complex as a multi-function calculator or computer software. Tools should be integrated into all tasks of teaching, learning, and assessment whenever appropriate. They allow the learner or the teacher to perform the following tasks:

- Represent and manipulate conceptual ideas in concrete or visual form (drawing, modelling, and simulation software)
- Perform rote tasks and computations, to free up the learner's time for thinking and reflection (calculators, spreadsheets, microcomputer-based laboratories, word processors)
- Gather, organize, display, save, and manipulate information or data (microcomputer-based laboratories, geographic information systems, databases, statistical programs)
- Collaborate and communicate (locally networked computers, telecommunication links to sites outside of school, electronic mail and bulletin boards, multi-media presentational software that incorporates words, pictures, sound, video)
- Search and access diverse information and opinions from many parts of the world (telecommunications news groups and mailing lists, electronic library access, bulletin boards)
- Overcome or compensate for physical disabilities (print-to-voice communicators for visually or vocally disabled people, telecommunications devices for the deaf)

Adding technology to a curriculum will not automatically improve teacher effectiveness or student learning. Teachers, administrators and parents must be constantly evaluating the usefulness of technology and its application to goals of learning, rather than using technology as an end in itself. Educators need a great deal of professional development, training, and support both to learn the particular technology and to implement technologies effectively and confidently in their classrooms.

The cost of computer technologies also threatens to decrease access to underserved populations, and increase the distance between classes, races, and genders. In order to develop equitable technology programs, communities must consider how to bridge the differences in economies of different districts, and challenge the pervading beliefs that technology is a frill or a privilege to use only after one has mastered the basics.

Mathematics and science instruction should emphasize connections within and across disciplines.

Problems in the real world rarely can be categorized as belonging to only one of the academic content areas such as mathematics, science, social science, or language arts. Most problems are complex, requiring the integration of a variety of knowledge and processes. Making connections among subject areas is a long-term goal for schools in Massachusetts.

In most schools today, mathematics and science are taught as separate subjects. This structure may permit the in-depth and comprehensive study of each subject. However, students also need many opportunities to work on problems that are more like real world problems, problems that cut-across subject areas. Interdisciplinary problems allow students to make connections among content areas, and can enrich their understanding of how mathematics and science are used in the real world.

While both mathematics and science courses typically include some problems that cut across other disciplines, each course emphasizes its own discipline. For example, some high school algebra courses include problems involving the laws of motion to demonstrate an application of quadratic equations, but the underlying science principles (in this case, the laws of motion) are rarely discussed. In science, students may use mathematical formulas as tools, but rarely discuss why the use of particular mathematics makes sense when investigating a scientific phenomenon. Effort is needed

to make more meaningful connections between mathematics and science, as well as other disciplines.

There is also a need to create connections within a subject area—to connect the three domains of science (life, physical, and earth and space) and to connect the different areas or standards within mathematics. In the elementary schools where one teacher is responsible for teaching all subjects, the classroom structure lends itself to interdisciplinary activities. In the middle and high school, where subjects are departmentalized, teachers need time to plan jointly and to expand their expertise in subjects not familiar to them. Chapter 3 emphasizes the importance of and suggests strategies for interconnecting concepts within mathematics or science and technology.

One area particularly suited to cross-disciplinary applications is statistics: collecting and analyzing data. Many aspects of students' daily lives reflect their need to work with and understand data, from understanding the probabilities of contracting particular diseases as a function of life-style choices to surveying students' opinions on a school issue and summarizing them to present to the principal. In science, data is the evidence on which theories are built. The way that data are defined, collected, organized, and represented will influence how people make sense of charts and statistics that daily newspapers and broadcasts use to summarize complex social situations, and to influence consumers and voters.

Assessment should be used as a tool to improve instruction and enhance student learning.

If learning can be seen as a journey, and teaching as a series of choices about schedule, itinerary, road, and vehicle, then assessment lets teachers and students know where students have been, where they are along the way, and where they might go next. The goal of assessment is to provide teachers with information about students' evolving understandings, skills, and knowledge, so that they can give feedback to students, and together make decisions about where to go next in the learning.

Scientific and mathematical inquiry is an active, not a passive, process. Assessment strategies used by Massachusetts teachers need to be congruent with principles of learning and teaching in mathematics and science. Assessment tasks that support the view of the learner as an active constructor of meaning allow students to demonstrate skills such as analysis, inference, comparison, and evaluation,

Pioneer Valley Regional School: Making Connections Work

One of the ways that Pioneer Valley Regional School (Grades 7–12) has created common planning time is through the use of early release days, which increased to one per month when the district expanded. Teachers asked the administration to be involved in planning the use of the release days. Ultimately the district planned for half the release days and the teachers planned the other half. Teachers worked within their departments to project a plan for the year. The work includes new course development, staff development, and interdisciplinary work with other departments. The interdisciplinary work might occur between two teachers from different departments, or two departments such as Math and Science working together to explore and develop connections, or several departments developing a thematic project.

At our school, most of us teach both junior and senior high classes in the course of the day. We also have a rotating schedule and heterogeneous class groupings. When the science and math teachers began to look at more interdisciplinary work, they decided to succeed despite the rigid schedule. They both had the same students over the course of the combined unit that involved using the school grounds to collect data for a science project, and the students then needed their math to organize, graph, and analyze their data. For the duration of the project, the students brought their math and science materials to both scheduled classes. One day the math teacher would be the contact person in the two adjoining classrooms, while the science teacher was out on the grounds with the data gatherers. On another day, the teachers would switch. On any given day, the students felt comfortable asking math or science questions of either teacher. If you asked students if they were doing math or science, their answer would simply have been, "Yes!"

Example: Life Sciences and NCTM Standard "Patterns, Relations, and Functions"

A group of first-graders is studying life at the ocean, and visits the beach to collect shells. Back in the classroom, students sort the shells in different ways: by characteristics of bumpy/smooth, color, shape; and then by families such as clams and snails. They make frequency graphs in order to answer the question: What are the most common kinds of shells? Are they the same for every beach? What is the average size of each kind of shell? Or is there a range of sizes? Do the patterns change at different seasons? Why might that happen?

as well as content or procedural knowledge. Assessment supports learning when it is an integral part of the ongoing activities in the classroom, and provides continuous, non-judgmental feedback, as well as an opportunity for students to practice skills and apply what they have learned in a new context.

Assessing student progress in the classroom

To gain information about the multi-faceted nature of a student's understandings, skills, and development, teachers need to gather multiple types of data and use several different strategies for assessment. This will also allow teachers to gain insights into areas of a child's strengths and weaknesses, and to become aware of the particular types of learning situations in which that child is most likely able to succeed. Using multiple strategies for assessment respects all children, and provides support and opportunity to learners with special needs related to cognition, gender, language, or culture. All classroom assessment tasks should be free of gender or cultural bias.

Just as students should be active participants in their own learning, so should they be actively involved in assessing themselves. Student self-assessment is critical in enabling students to become responsible for their own learning. Students who are involved in the process of developing and applying criteria for acceptable work will more likely develop responsibility for their performance. What constitutes successful or acceptable work can become part of the ongoing process of reflection as students are working on a problem. Students who are able to develop, apply, and internalize criteria for excellent performance can constructively build on their strengths while diagnosing and treating their weaknesses (Zessoules & Gardner, 1991).

Performance-based assessment

Particularly congruent with the goals of instruction embodied in these Frameworks, performance assessment is based on students' demonstration of their ability to use the skills they have learned and the conceptual understandings they have developed in the context of a complex problem. Characteristics of good performance assessment are that tasks: present non-routine, open-ended problems that are grounded in real world contexts; require students to further define the problem as well as construct a strategy for solving it; involve sustained work; may involve group and/or individual work; deal with big ideas and major concepts rather than discrete facts; are

broad in scope, integrating several concepts; and blend content and process (Baron, 1990).

The performance task should be a natural and integral part of the curriculum. Assessment itself, including the time it occupies, is not outside the curriculum but is an important part of reaching educational goals (Collison, 1992). As is the case with any assessment technique that promotes understanding, timing should be flexible. As pointed out by Stenmark (1989, p. 32), "Timing performance tends to discourage persistence and to promote thoughtlessness and jumping to conclusions. Quickest does not mean most talented."

There are numerous methods of performance-based assessment; teachers may well choose to use various combinations for purposes of evaluating the multiple dimensions of students' progress and understanding. Three such methods are described below.

Open-ended written assessments

Students can be asked to respond in writing to open-ended questions and problems. These assignments, which are produced by students as part of their regular work, can provide extensive insight into students' thoughts. Paper and pencil tests can also be expanded to include open-ended questions that can be solved a number of ways, allow for creative expression, and encourage comparative analysis and reflection. These written assessments can ask students to communicate their strategies, hypotheses, or solutions using writing, graphs, or drawings. For teachers who are most comfortable with traditional paper and pencil testing, this may be a comfortable place to begin bringing assessment in line with a more inquiry-based approach to science and mathematics.

Portfolios

A portfolio is a collection of an individual student's written work and products (including videotapes, charts, models, constructions). When using this assessment method, the student and teacher decide what work to include: the work that best shows the student's abilities and strengths, the work that is most representative of his or her efforts, and/or the work that shows the student's progress over a period of time. When using the portfolio as a method of assessment it is critical that student and teacher work together in establishing criteria for selecting work as well as criteria for judging merit.

Observation

Observation is perhaps the most frequently used method of obtaining information for purposes of assessing student understandings. Informal observations can make note of behaviors that occur spontaneously within the natural flow of classroom events. Formal observations are pre-planned, target specific skills, and refer to specific criteria. The criteria for performance (for example, poor to excellent) are determined ahead of time and made known to the student who is being observed.

Relationship of classroom assessment to grades and external assessment

Decisions about how to integrate new strategies and theories of assessment into current school practice and the requirements of standardized testing will have to be negotiated as the principles in these Frameworks are adopted over time. It is critical that standards for classroom assessment, the assignment of grades, and external assessment are congruent with each other and with the principles outlined in these Frameworks. In general, the ways grades are generated and assigned need to be meaningful to both teachers and students; they should represent what children can do with information, not how well they can memorize facts. While grades are most frequently associated with classroom assessment, they have the potential of suppressing risk-taking and cooperation, and discouraging learners who are struggling (Berger 1991).

Those who developed these Frameworks feel strongly that more work must be done in Massachusetts to bring statewide and external assessments more into alignment with these principles and standards. Changing the nature of assessment tasks will also require support and professional development for teachers, as well as shifts in school- and community-based goals and systems.

Chapter 3:
The Discipline and Vision for the Frameworks

Mr. Walo teaches 3rd-grade science in western Massachusetts. He is a member of a team of three teachers (himself, a teacher of mathematics, and a teacher of life science) which has planned that the students, working in groups, will drop hardboiled eggs onto the floor from different heights, this in order to study force and motion and matter and energy. Why hardboiled eggs? Because they are tough and can do considerable work on other objects before they crack (with a delightfully scrunchy sound); because they can lead to connections with other disciplines such as life science, art, and health; and because they—with parental consent—can be eaten afterward as a treat. While the students watch, Mr. Walo takes an unboiled egg from a carton and (intentionally) fumbles it so that it drops to the floor and splatters magnificently. "It was supposed to have been boiled!" he shouts. "Well, that shows you that scientists make mistakes like everyone else, and that's OK as long as they don't try to hide it." As the squealing and cleaning fuss quiet down, questions begin tumbling in: Why?

"It's about matter and energy," Mr. Walo says. "We're going to use hardboiled eggs to see the energy they have when they're moving, for instance when they fall. We're going to find out how much work an egg can do on something else before its shell cracks. But first let's go to our Roundtable and decide exactly what we want to do and how we will do it and what measurements were going to make and who will record what—there is a lot to do, but if we all plan it carefully—do you think we can hack it, team?"

*"Yes!" ...**

The DISCIPLINE: *"Science is systematized knowledge derived from observation, study, and experience. Science is verified knowledge; that is, knowledge that can be validated and communicated to other people." (George Simpson) "Science is a refinement of thinking of daily life." (Victor Lenzen) "Science is such knowledge as hath a tendency to use." (Robert Boyle)*

This is a part of The Vision:

The Learners, the students as think-tank, enabled not only to ***learn science*** at every level but also to ***be scientists*** at every level, and make real-world connections.

Discipline and Vision

THE DISCIPLINE: It is the actions used in developing, producing, using and assessing artifacts. This view is the broad descriptive. It suggests that technology is a body of knowledge and action. It is used by people to apply resources in developing , producing,, using and assessing products, structures and systems. It extends the human potential for controlling and modifying the natural and human-made environments. Technology is an entity unto itself with respect to science; nevertheless technology is applied science. Nor are Science and Technology independent social phenomena. "Hindrance of the advancement of learning hath been because thought, theory, and practice have always been divided." (Sir William Petty)

Ms. Chen teaches an 8th-grade science class in Northeast Massachusetts and has joined with a teacher specializing in technological design to mount a project-based unit; in particular the unit incorporates designing against physical shock to delicate systems when they are transported, in this case raw eggs as they fall to the floor. Why eggs? Because in the unit on Transportation, the students must devise a cradle to protect a fragile object; because if they know that they must clean up the mess of an unsuccessful experiment, their motivation to do well is heightened; and because they can lead to connections with other disciplines such as life science, art, and health. Ms. Chen lifts one of six eggs from a carton and announces, "I'm going to drop this on the floor." Instant silence, but then: Why? Ms. Chen says, "You are also going to drop eggs, but in cradles that you invent and design and build out of the materials here on my desk. And you are going to be right beside your cradle when you drop it—get the picture? Here goes!" she shouts as she lifts the egg and lets it fall. Amid screeches and scrambling from the expected mess, the egg drops and makes a thud as it cracks. "Disappointed?" she asks. "You assumed it was fresh, but it was boiled—remember our discussions about assumptions in everyday life and especially in science and technology! The eggs that you drop will be fresh however, and your cradle—hopefully—will keep them from doing the Humpty-Dumpty thing when they hit. Discuss your ideas with each other. Give and take, but be sure you credit whose idea it was if you use it...."

This is a part of The Vision:

The Learners, the students as an R&D group, empowered not only to **study technology** at every level, but to **do technology** at every level.

Discipline and Vision

A 3rd-grader asks, "Mr. Walo, may we throw the egg instead of dropping it?"

He replies, "You could, but your throw would be a little different each time, and we would need to keep your throw the same each time to make our measurements mean anything. We can depend on gravity to be exactly the same all the time. If we use gravity and each egg weighs the same, then the only changes in what damage the egg can do when you drop it will be because you dropped your eggs from different heights...."

An 8th-grader asks, "Ms. Chen, who needs to know how to cradle an egg so you can drop it on the floor, when nobody is going to drop an egg on the floor on purpose anyway? It's dumb!"

*Ms. Chen replies, "You never know—somebody might be dumb—or smart—enough to want or need to know how to cradle an egg for dropping it on the floor. Anyway, the cradle you design might lead to ideas for other things, like cradles for people for escaping from....**

<u>This is part of The Vision:</u>

The Learners, the students as inquirers, drawn into making meaning of the natural world as if they are pulled by a magnetic force.

** Vignettes supplied by J.W. Coleman, from conversations with teachers. Names are fictional.*

Discipline and Vision

A 3rd-grader gives an opinion. "Mr. Walo, that won't work." says Billy. His classmate offers a counter-opinion. "I think it will. See?" says Jimmy.

"Well, lets look at it in more detail," says Mr. Walo to his 3rd-graders, "and talk it out. Come on, let's go to our Roundtable."

<u>This is part of The Vision:</u>

The Learners, the students as debaters, spurred to critical thinking and reasoning for themselves.

....

An 8th-grader asks, "Ms. Chen, how many times do I have to drop it from each height?"

Ms. Chen replies, "Well, let's decide together—it's up to you and your data. How reproducible is the data for, say four feet? And what were your independent variables? Record your predictions in your journal and be reflective in your thinking...."

<u>This is part of The Vision:</u>

The Learners, the students as recorders, urged always to collect data, make predictions, generate hypotheses, test them, and retest them as necessary.

Discipline and Vision

What Conjures The Vision and Gives It Its Foundation:

A 4th grade class in Quincy is trying to determine optimal growing conditions for bean plants. In small groups, students brainstorm and try out different theories, measuring and testing variances in amounts of water, sunlight, and soil composition. Different group members organize the data collected into charts and graphs; others record the group's findings in a journal. The groups present their results and predictions to the class.

Compare the work in the class in Quincy to the work in a laboratory in a business environment in the Greater Boston area:

A research team at a photography company is trying to find a more environmentally-friendly package for their film. The team brainstorms ideas, uses on-line computer services to research information on various materials, develops prototypes using the different materials, and conducts focus tests with customers. The team calculates expected development costs, and projects the impact on sales. The team then presents its findings and recommendations to upper management of the company.

....

In the above comparison, students in the fourth grade are behaving in ways similar to those of the professional research team: investigating a question that requires them to experiment, propose and test hypotheses, and record, interpret, and present their findings to others. The classroom above epitomizes the **Vision**; the Learners are actually doing science and technology, actually being scientists.

Such a vision is in keeping with, and arises inevitably from, a visualization of the goals expressed in the Massachusetts Education Reform Act, the Massachusetts Common Core of Learning, the proposed National Science Education Standards, Goals 2000, and in dialogues with Massachusetts business and educational leaders, administrators, parents and the teachers in the public schools of our Commonwealth. The Vision is thus an articulation of our collective hopes for our Learners as they are educated in the Commonwealth.

Discipline and Vision

Reflections

Teachers may ask 'what other kinds of questions could you ask about eggs?'

Mr. Walo says to his 3rd-graders "Let's find out about the egg itself a little more...."

Ms. Chen says to her 8th-graders, "We have been concerned only with protecting the egg. Aren't you curious about the egg itself? I know I am...."

An 8th-grader remarks: "I read a science story about this egg on another planet. It was made out of titanium - could that be real?"

Another 8th-grader says: "Most liquid stuff boils when you heat it, and most solid stuff melts. Why does the egg get solid inside when you heat it in water instead of just boiling and blowing up like it does in the microwave?....

A 3rd-grader announces: "I saw this picture of a beautiful diamond-and-pearls egg made by an artist for a Russian Princess...."

Another 3rd-grader asks, "Can this egg still hatch into a chick? I'm going to write a song for it! I asked my ballet teacher if I could be a chick hatching in the next recital."

An 8th-grader wants to know "Did a fight ever happen over eggs? I mean a big fight—like a war? Are eggs good for you or is there too much cholesterol?"

Practicality and Practicability Of The Vision:

The practicality of the Vision is that the Learner is at center at all levels of schooling, while the teacher, though indispensable, is peripheral as facilitator and mentor. The Learner will always be nudged, rather than talked at, to "apply the fundamental principles of the life sciences, physical sciences, earth/space sciences and the science of technology to analyze problems and relate them to human concerns and life experiences."(1) The Learner will come to experience and understand distinctive features of science and technology, as well as the commonalities existing between these disciplines and all other disciplines. The Learner will become questioner and answer-finder (with the teacher as facilitator). The

(1) The Massachusetts Common Core of Learning, July 1994, page 6.

Discipline and Vision

Learner will become solution-poser (with the teacher as problem-suggester).

Realizing the Vision for Science and Technology education in Massachusetts is entirely practicable within the umbrella concept of interactive, interdisciplinary, experiential education (IIEE), i.e., hands-on, cross-subject-lines, with student-teacher dialogue and questions at any time, which is already in place, either in part or totally, in many districts of the Commonwealth.

Guiding Principles

Guiding Principles for Framework Construction:

A Chinese proverb translates as "Learning Has No End." This belief and the tenets discussed in Chapter 2 under *Principles for Learning, Teaching, and Assessment* are primary foundations for the construction of this Framework. Additionally, nine other Guiding Principles have been used:

- **All students should be enrolled in integrated science and technology classes every year from Kindergarten through grade 12.** Science classes from K-12 be heterogeneously grouped and non-tracked. These classes should encourage problem solving and thoughtful collaboration and should foster skills of scientific inquiry and technological design. In grades K-10, students should be enrolled in classes which teach all domains of science (life sciences, physical sciences, and earth and space sciences) every year.

- **There also should be no *tracking* in grades 11 and 12, but a broad selection of Science and Technology elective courses should be available in every school so that the individual student can design a tailored pathway geared to his/her needs, for example, leading to entry into nursing school or a specific job after graduation.** These pathways (2) should be selected and/or designed by the student with guidance from teachers and parents, but should not be labeled or discernible as any particular track in school records. The menu of possible courses for selection should include: classes preparatory for post-secondary study in physical sciences, engineering and technology, life sciences, bioengineering and biotechnology, agriculture, and computers. Courses for entry into the workplace and internships and/or work-study opportunities should also be provided.

- **Education in Science and Technology should emphasize the *quality of understanding* rather than the *quantity of information presented*.** Teachers are encouraged to provide renewed emphasis in this area.

- **Students learn science and technology by engaging in authentic tasks of inquiry, reasoning, and problem-solving that reflect real-world practice.** Teachers are encouraged to present constant opportunity for such engagement (note Vignettes in the Content Standards section of this chapter).

(2) The High Stakes of High School Science (The National Center for Improving Science Education, 1991)

Guiding Principles

- **Learning is a process of an individual actively constructing his/her own understanding of the world through direct experience. Hands-on and inquiry-based experiences can deepen the understanding of abstract concepts through encouraging the usage of process skills, communication, and reflective thinking.** Teachers are encouraged to present constant opportunity for such experience (note Vignettes in the Content Standards section of this chapter).

- **Learners can benefit through the social and organizational skills developed by working in groups.** Teachers are encouraged to make such groupings, but without the taint of "Tracking" and yet with the concerns for equity, including language barriers, physical handicaps, resources, gender, cultural diversity and other equity issues. These concerns may pose dilemmas and difficulties, but the teacher is encouraged to pursue the ideal with diligence.

- **Education in Science and Technology should emphasize connections within and across disciplines.** Teacher collaboration and/or team-teaching at all levels is encouraged; preparation time for establishing and maintaining such teacher partnerships and shared teaching units and projects should be regularly scheduled.

- **A teacher of technology should be a permanent member of a teaching team.** Technology is an integral part of our social structure, defined in part by its use to transform the environment, the ideology, and the sociological elements of our Commonwealth and nation.

- **Habits of mind for promoting successful processing of learning and work in science and technology should be introduced to the Preschool and Kindergarten student and reinforced at every level so that the habits eventually become internalized.** Teachers are encouraged to emphasize curiosity, skepticism, receptivity, environmental stewardship, honesty, objectivity, tolerance of ambiguity, open-mindedness, and acceptance of criticism by their peers. Also, it is hoped that all teachers will develop an awareness of, and a sensitivity to, differing social and cultural habits of mind, which go along with the cultural diversity in the student population.

Framework Organization

Organization of the Framework

Basis for the Organization --

The last few decades have been characterized worldwide by increasing interaction between Science, Technology, and Society. As the results of scientific research and the products of technological design have become more influential and available, both individual and societal awareness of their interrelationships and effects in everyday life have constantly grown. Yet, even as the world confronts rapidly expanding social, environmental, and other types of problems, surveyors of science and technology programs in schools show that the teaching has remained mostly in the traditional abstract, decentralized fashion. In most systems, there has been little incorporation into science and technology curricula of the interrelationships either with other disciplines of study, or with just being a part of society. A way to rectify this situation and bring (and keep) science and technology education abreast of the times has been suggested by the National Research Council (NRC) of the National Academy of Science.

As a result of a development effort involving science educators, scientists, technology educators, parents, and policy makers, the National Research Council in 1993 originally conceptualized the totality of Science and Technology CONTENT as being in four areas categorized as **Inquiry**, **Subject Matter**, **Technological Design**, and **Human Affairs**. Massachusetts has embraced these four original NRC-defined categories as the organizational scaffolding upon which to build the Massachusetts-specific Science and Technology Framework, with the renaming of two of the areas: from **Subject Matter** to **Domains of Science**; and from **Technological Design** to **Technology**.

- **Inquiry** encompasses the development of the necessary skills such as observing, hypothesizing, designing experiments, and interpreting evidence, within the context of solving problems in everyday life rather than being employed as mere rituals of some abstract 'scientific method.'

- **Subject Matter [Domains of Science]** encompasses the presentation of the principles and laws of natural science and technology embedded always in realistic contexts, and portrayed with down-to-earth illustrations, which the students can either experience directly or imagine easily.

Framework Organization

- **Technological Design [Technology]** focuses on how technology contributes to solving human problems or meets human needs, both as a basis for placing technology in its proper relation with science as a means for providing solutions, and for providing students with practical activities including design challenges.

- **Human Affairs** focuses on the social consequences of science and technology in day-to-day living, always within the context of the impact on total society and the relevance to the individual students' lives.

These four areas are conceived by the NRC not as monolithic but as interdependent:

> *"It is important to recognize what these categories [content areas] represent and - equally importantly - what they do not represent. They serve to organize and group four overlapping "clusters" of student learning in science. We expect that effective science curricula will routinely interweave these important aspects of what students should know and do in a variety of ways.*
>
> *By contrast, these content categories [content areas] do not imply that separate science teaching units or courses should support each topic in isolation. The "things" organized under these headings are intended attainments of students, not the instructional experiences through which students develop such understandings [and skills]. In other words, the organization of standards for science content is not necessarily congruent with the organization of a particular program of study or science curriculum.* (National Research Council. July, 1993)

Massachusetts embraces both the four originally NRC-defined categories of science and technology content, and the necessity for their interdependence and overlap as a requisite for producing reformed and more meaningful curriculum. The NRC organizational scheme is thus the basis of organization for this Framework.

Content Areas

The Massachusetts Specific Science and Technology Framework

The relation of the four original NRC content areas can be interpreted in terms of four areas constituting a Euler (Venn) diagram, with the desired Chapter 3 of the Science and Technology Framework arising from the intersection of all four areas (Diagram 1). The development of the Massachusetts Specific Framework was analogously conceived as being driven by the four areas (Diagram 2).

Content Area #1: Science as Inquiry

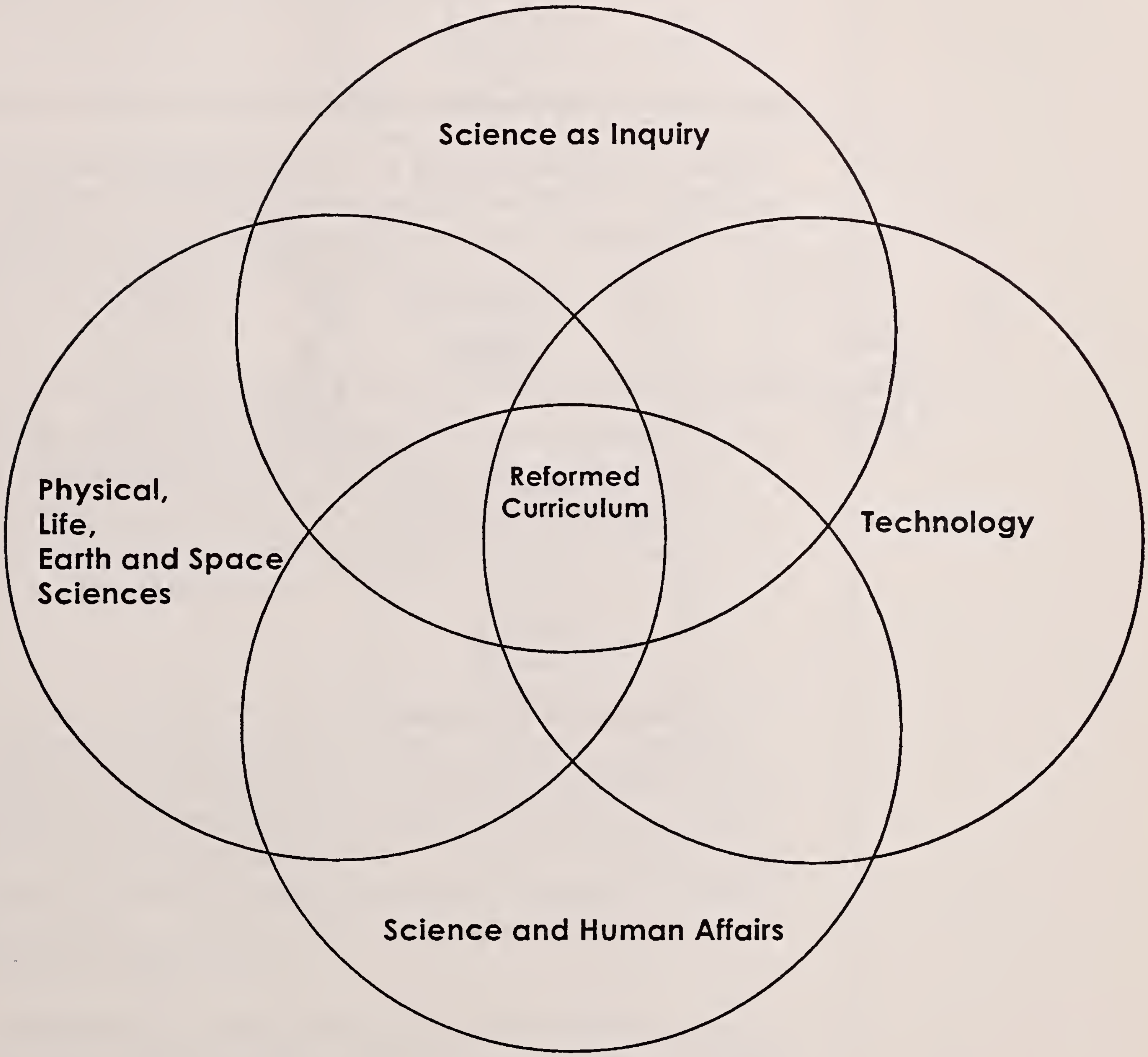
> *Science programs should help students to answer the questions of science, not be presenting assertions or authority-determined answers, but by allowing* [students] *to propose and pursue the ideas, concepts, and information.* (National Science Teachers Association, 1992, p.3.)

Before there was science there was inquiry; the knowledge and practices people have come to know as 'science' have arisen from human curiosity about the natural world and the desire to seek information about that world. Active inquiry in the classroom can capitalize on that natural curiosity of students and can engage them in inquiry processes as actually practiced by scientists, i.e., the students can participate in series of creative, systematic procedures that lead to a deeper understanding of natural phenomena and related concepts and theories.

Learning through inquiry requires an environment that fosters scientific "habits of mind", which are the attitudes shaping the way people approach solving a scientific problem, the investigational methods they use, and their methods of analyzing findings and interpreting results. Skepticism, receptivity, objectivity, honesty, and curiosity are qualities that are central to learning through inquiry. In Massachusetts schools, therefore, it is most desirable that inquiry be a critical component of the science curriculum across all domains of science, and for all students regardless of their academic proficiency. The emphasis on scientific inquiry should begin in early childhood and extend through secondary school.

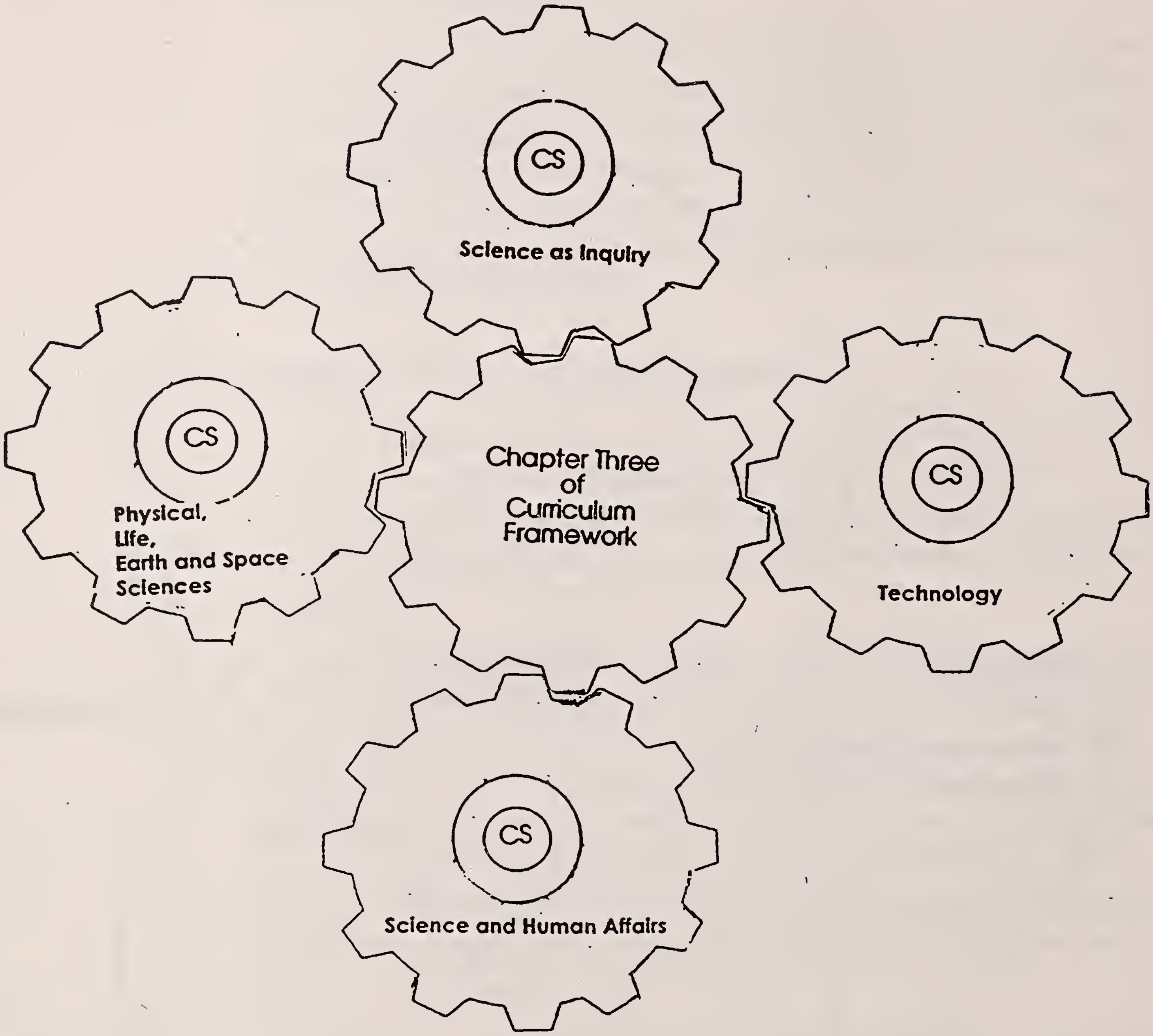
In science classrooms that stress inquiry, students engage in projects and investigations in which they combine thinking and

NRC
Science and Technology
Content



Euler (Venn) Diagram of the four NRC Content Areas

Diagram 1



CS = Content Standards

Diagram 2

Content Areas

doing as they use their inquiry skills. Students, like scientists, do not always use their inquiry skills in any particular predetermined sequence, nor do they use their inquiry skills as isolated elements in the overall questioning process. In real classroom situations, inquiry skills and knowledge of subject matter are inextricable. Inquiry-based, hands-on skills, and cognitive processes should be taught within the context of the domains of science.

Content Area #2: Science Subject Matter [Domains of Science]

There is a knowledge explosion of unprecedented proportion in the field of science. Nowhere is the problem of too many topics, and too little time to develop a deep understanding of these topics, more prevalent than in the field of science education.

> *"As the body of knowledge developed by the scientific community has grown, science textbooks and programs have become increasingly overburdened by vocabulary, facts, and information, while remaining generally unresponsive to growing knowledge of how conceptual understanding of science is developed in young people."* (NRC, Feb. 1993).

In order for students to develop meaningful understanding of the concepts of science, they need to have extended opportunities for in-depth inquiry because they construct their understanding by immersing themselves in extensive inquiry around particular concepts. This fact has provided the field of science education with the challenge of addressing a difficult, yet essential question: *What are the most central concepts, laws, and theories that we want our youth to understand and be able to use?*

Traditionally the domains of life, physical, and earth/space science have been the basis for organizing science curricula, and the three domains have been studied separately by the students. At the secondary level, in particular, students usually study one domain each year, a way of organizing curriculum which has come to be known as the "layer cake" approach: typically, earth/space science in the 8th grade; biology in the 9th or 10th grade; chemistry in the 11th; and physics in the 12th. For the most part, there is little if any

coordination of what is taught from year to year. Furthermore, approximately three-quarters of high school graduates in the U.S. stop taking science after the 10th grade, which means that many of our students are graduating without having taken chemistry or physics. Thus science education reformers in the U.S. are asserting "the time has come for the...layer cake to be dismantled" (NSTA, 1992), because when the domains are taught in isolation, students may perceive that concepts in the domains are only vaguely, if at all, related to each other. Connecting of the domains of natural science with each other and with mathematics, and where possible and applicable, with other disciplines should be the goal of the teacher.

Education in science and technology should be structured so that students develop a comprehensive understanding of concepts, e.g., energy is energy is energy, whether in one domain or another. People who consciously use and practice science rarely draw from one domain in isolation from the others. Massachusetts schools should mirror the realities of how people actually make use of scientific knowledge and skills, e.g., oceanographers use their knowledge of physics, chemistry, biology, and earth and space sciences when studying organisms in a tidal pool. Designing curricula that help students make important connections can be a complex undertaking for schools, but the desired organization of instruction across the domains can be done in a variety of ways, ranging from well-coordinated "domain-based" approaches to fully "integrated" approaches.

Coordinated Domain-Based Approach

A coordinated domain-based approach is one in which separate domain subjects are taught every year, but while each unit focuses on its Content Standards, the curriculum is structured to help students make connections across to the other domains.

In a well-coordinated domain-based science program for eighth graders, for example, students might study the physical properties of light when studying physical science. They could investigate how the eye works when studying life science. And, students could explore how the moon reflects light from the sun when studying the earth and space sciences.

In a coordinated domain-based program for 10th graders for example, light might be studied as it pertains to geometrical optics, image formation, and rudimentary spectroscopy with telescopes in earth/space science. In another example, 12th graders might study the effect of radioactivity on cellular material in a class in biology,

Content Areas

while studying the causes of radioactivity in chemistry or physics, while studying the uses of radioactivity as a way to determine the age of rocks and fossils in earth/space science classes.

Integrated Approach

In a curriculum which integrates the domains, the course of study affords opportunities for students to learn and associate key concepts without the constraints of a model which isolates each domain. A school district, for instance, might focus on a concept such as *Change and Patterns,* and as an example of this integrated approach on the early elementary level, might select *the seasons.* In this instance, the children could record changes they observe in deciduous trees, research animal hibernation, look for animal tracks and observe how animals gather food, study changes in plant growth, keep track of the sun's position in the sky, keep a log of sunrise and sunset times, and explore how water in the natural environment changes state from season to season. Another example, this time under the concept of *Interaction and Systems* and at the middle-school level, students might focus on a river environment and test the chemistry and flow rate of the water, the geological formations in the river bed and along the banks, and the river environment and ecosystem.

Content Area #3: Technological Design [Technology]

Today we live in a human-designed and human-built world that is constantly changing. Each of us is surrounded by a technological environment which controls nearly everything we come into contact with, including the information we receive, the products we use, and the speed at which we travel. History is a record of people adapting to the natural and human-made environment; and a story of inventions and innovations, tool development and materials mastery, and the constant struggle for progress and a better quality of life.

Children take naturally to technological design and usage (witness programming of the VCR). Young children are veteran technology users by the time they enter school. They ride in automobiles, use household utilities, operate wagons and bikes, use garden tools, help with cooking, operate the television set, and so on. Children are also natural explorers and inventors, and they like to make things. (AAAS Benchmarks). It is important to capitalize upon these

Content Areas

experiences and natural inclinations. Learners should have opportunities to identify problems of human adaptation to the environment and then to apply problem solving strategies, to design investigations, and to build models and tools to solve the problems. Investigating the salinity of the water in Vineyard Sound is science; creating a way to make the water drinkable is technology. Science seeks to understand the world and the problems posed for its inhabitants; technology attempts to analyze and solve the problems so posed. Thus science and technology are intimately bound; they coexist and interact. Technological design should therefore be integrated with science to be a part of the same learning experience for students throughout K-12.

The content area of Technology emphasizes the importance of creating solutions through the process of design. The design process affords the opportunity for identifying a problem, applying knowledge and skills for conceptualizing a solution, weighing options, making decisions, constructing and testing tools, reflecting on results, and reformulating ideas as necessary.

The content area of Technology is divided into two main areas: Human Generated Technology, including *Communication, Construction, Manufacturing, and Transportation;* and Processes of Technology, including *Power Technology, and Bio-related Technology.*

- **Communication** technology uses technological design to connect a source of information (the sender) with a receiver by means of a *communication channel.*
- **Construction** technology uses technological design to produce and erect generic *structures* at or on a site.
- **Manufacturing** technology uses technological design to change the form of *materials* to increase their worth or usefulness.
- **Transportation** technology uses technological design to *relocate people and/or cargo* through vehicular and non-vehicular systems.
- **Power Technology** uses technological design to *convert energy* into different forms or products.
- **Bio-related Technology** uses technological design for *producing process techniques* relating to *agriculture and biological processes* tied to fuel and materials production.

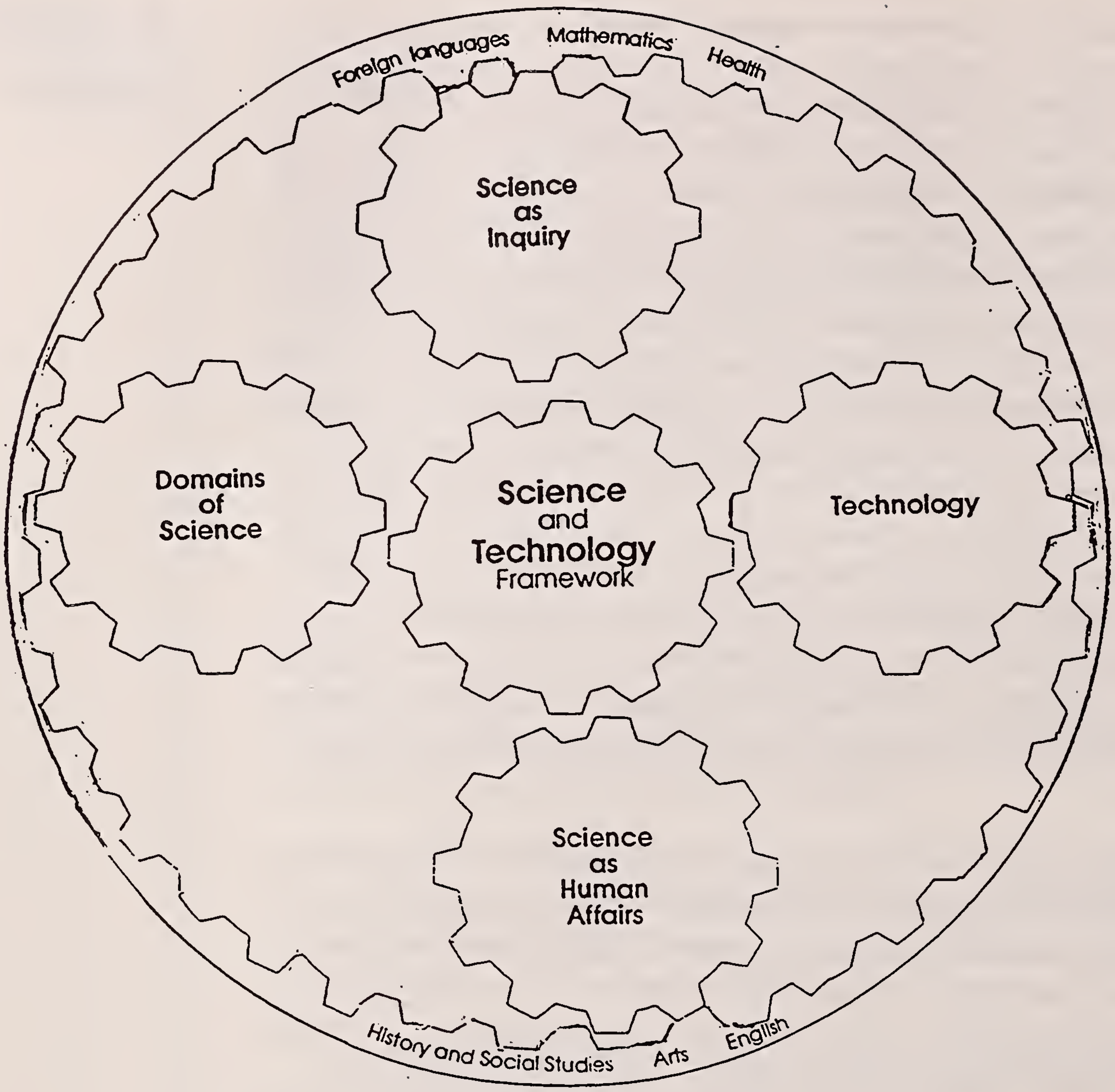
Content Area #4: Science as Human Affairs

Content Areas

As a human endeavor, science has changed people's outlook on the world and has inspired communities and nations to set new goals and aspirations. Throughout history, scientific knowledge has played an important role in many aspects of life and culture; scientific theories have helped shape our understanding of the world and of our impact on the environment. Advances in science and technology such as the ability to circumnavigate the globe and land a person on the Moon have changed the way we view ourselves and our place in the universe, and have led to the development of technologies whose applications have had profound effects on every day life. Three hundred years ago, three-quarters of the people in the United States needed to toil on the land just to provide the food the people needed. Through science and technology, only one hundred years later it required only one-quarter of the people to toil on the land to produce food for all, even though the population had grown. Today less than five percent of the population is involved in agricultural production.

During the early twentieth century, technologies were developed to make labor at work and in the home less arduous. Simultaneously, better sanitation, improved water supplies, and medical advances such as the development of antibiotics helped people to live longer and healthier lives. Today our lives are changing rapidly due to microchip technology and the new ways of communicating electronically. But increase in knowledge in science and technology has not always meant progress. Many people are concerned about the effects of the unbridled use of science and technology on the quality of life, for example the development of weapons capable of mass destruction of both people and the land. The exploitation of natural resources had repercussions which were not taken seriously for many years.

Hopefully we are learning from our past mistakes. While we are naturally inclined to use science and technology as a means for making our lives more comfortable, those very conveniences can exact a heavy toll on the environment and on us. Our cars enable us to travel easily, yet pollute the air we breathe. Pesticides afford greater crop yields, yet they may produce irreversible damage to the ecosystem. Underground tanks conveniently store petroleum, yet their leaks contaminate our drinking water.



The Science and Technology Framework is driven by:

- Science as inquiry
- Domains of Science
- Science and Human Affairs
- Technology

and

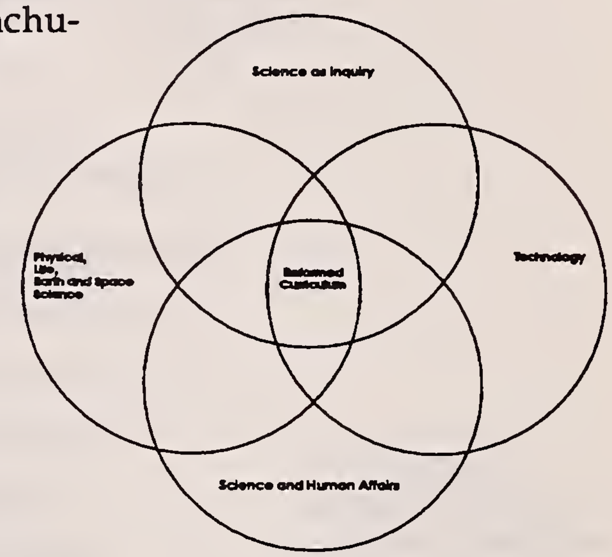
- Mathematics
- English
- Arts
- History and Social Studies
- Foreign Languages
- Health

Drivers for the Massachusetts Science and Technology Framework

Diagram 3

Content Areas

From the above considerations, we suggest in this Framework the working approach which treats human affairs as inextricable from science and technology subject matter and inquiry, rather than presenting human affairs as an afterthought at the end of particular curriculum units. By embedding scientific ideas in the context of human affairs, teachers can more clearly establish a 'need to know' and a higher level of student interest and motivation. In Massachusetts, we feel that an important way of motivating the interest of students from sections of the population that have been historically under-represented in the sciences is to situate content areas in appropriately chosen political, historical, and cultural contexts.



Each **CONTENT AREA** presented here is in every case to be understood as interrelated with others. This general concept is shown in a specific example for a topic "Effects of Oil Pollution on Ocean Ecosystems." (Diagram 4)

Finally, in this Massachusetts Framework, the four content areas discussed above not only intersect each other but also are conceived as connecting to content areas in other disciplines, i.e., mathematics and all other disciplines taught in the Massachusetts schools, as is shown in Diagram 3. Thus input and output from all Frameworks should interconnect with the Science and Technology Framework.

Content Standards

Science and Technology Content Standards

A main purpose of the Massachusetts-specific Science and Technology Framework is to present Content Standards, which comprise the essential elements of subject matter which students should know and be able to use at a given point in their science and technology education. We embrace the concept of Content Standards and encourage that where possible, districts should develop and design curricula which place the Content Standards in contexts that address current issues, resources, and environments in the Commonwealth. In the organization of curricula, Content Standards should be incorporated at various grade levels and in a variety of ways so that students can deepen and flesh-out their comprehension of the concepts over time. These Content Standards proceed from considerations of the four Content areas discussed above.

The Massachusetts-specific **CONTENT STANDARDS** for Science and Technology are divided into four grade-groupings: PreK-4; Grades 5-8; Grade 9 - 10; and Grade 11-12 . Although the **CONTENT STANDARDS** are delineated in this Framework according to the

Science Subject Matter Domains

Students develop understandings about properties of matter (physical sciences); characteristics and diversity of organisms (life sciences); and interactions and cycles, and the earth in the solar system (earth and space sciences) as they investigate some of the sources of oil pollution and its effect on ocean ecosystems.

Science and Human Affairs

Students consider the environmental consequences on the oceans of the decision industrialized countries have made to use oil as a major source of energy.

Science as Inquiry

Central Topic: Effects of Oil Pollution on Ocean Ecosystems

Students pose questions to investigate.

Such questions might include:

- Where does oil pollution in the oceans come from?
- How does oil pollution affect ocean ecosystems?
  - -estuarial ecosystems?
  - -coastal ecosystems?
  - -deep ocean ecosystems?
- What are the biological effects of oil,
  - -on aquatic organisms (both plants and animals)?
  - -on sea birds?
- How is the dispersal of oil affected by wind and currents?

Students plan and conduct investigations to explore these questions. During their investigations students make observations, and gather and record data.

Their work might include investigations such as: comparing a polluted estuary with an unpolluted one; measuring amounts of dissolved oxygen in water, studying interactions of oil and water, and detergents used to clean up oil pollution. Students' work might include, among other activities, making counts of plants and animals, and investigating food chains, loss of biodiversity, tides, ocean currents, and salinity. Students could record data from their investigations in their science notebooks.

Students analyze, explain, and communicate findings.

Students use both qualitative and quantitative methods of analysis; find ways, such as graphs and diagrams, to represent data; conduct small and large group science conferences; write reports; and present the processes and findings of their investigations to other people.

Diagram 4:

This diagram illustrates how the four science content areas (and mathematics) can be connected within a science unit for seventh graders. The central box shows the progression of the major work students do. While they are investigating their questions, students use mathematics, develop fundamental understandings in the three subject matter domains of science, relate their inquiry to societal problems and issues, and use technology not only as a tool in their investigation but also to develop possible solutions to problems they identify in their work.

Technology

Students identify a problem: How can we lessen oil pollution and its effects on ocean ecosystems?

Students design, implement, and evaluate a proposed solution to the problem. Students could consider alternative sources of energy which can be used to replace oil, for example, a solar heating system. Students might design and build a model of such a system and measure its efficiency.

Connections with Mathematics

Data analysis is a central part of students' investigations of the effects of oil pollution on ocean ecosystems.

Content Standards

indicated groupings, it is understood that learning is not so neatly compartmentalized, and that students develop at different rates and learn by having opportunities to "revisit" particular concepts and skills. Consequently, the grade level spans are intended only as a general guide; educators should use them flexibly according to the realities within their districts, schools, and classrooms. Yet, while the use of this Framework is not mandated by the Massachusetts Education Reform Act, assessment of students is mandated by the Act at grades 4,8 and 10. The Content Standards herewith presented are the targets from which the assessment tools will be created. An example is shown for **CONTENT STANDARDS** relating to the properties of matter as studied in physical science, for the four grade groupings (page 63 , Diagram 5).

Use of the Content Standards

There are many different ways for students to attain the indicated knowledge or capability at the prescribed level. *AS AN EXAMPLE ONLY*, it might be visualized that **CONTENT STANDARDS** arise from the intersections of 'big ideas' or 'themes' or 'strands,' say **CONCEPTS** and **CONNECTORS** in our example. Arbitrarily let us take six of each and define them to be for domains of science and for technology in all grades PreK-12; a chart might then be prepared as on page 62. (Diagram 6)

The Intercept of the CONCEPT "Interaction and Systems" with the CONNECTOR "Matter and Energy" then could be the context, i.e. in grades 9-10, for the Content Standards in Physical Science (Interactions of Substances, pg 100); in Life Sciences (Matter and Energy in Ecosystems, pg. 104); in Earth and Space Sciences (Matter and Energy in the Earth System, pg. 106); and in Technology (Resources and Technology, pg. 109). Cross-connections would also be established between the domains.

Content Standards

Domains of Science & Technology
Example for grades PreK - 12

CONNECTORS \ CONCEPTS	INTERACTIONS AND SYSTEMS	SCALE AND MEASUREMENT	ORGANIZATION AND MODELS	DIVERSITY AND ADAPTATION	CHANGE AND PATTERNS	FORM AND FUNCTION
MATTER AND ENERGY	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards
FORCE AND MOTION	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards
COMMUNICATION AND TOOLS	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards
ORGANISMS AND HEREDITY	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards
EARTH AND SKY	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards
ENVIRONMENT AND RESOURCES	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards	Content Standards

Diagram 6

Note: A Given content standard may arise at different intersections of concepts and connectors depending upon the curriculum developed by the school district.

*As the Chart shows, in this example the 36 intersections of the **CONCEPTS** and **CONNECTORS** became the Content Standards for the Domains of Science and Technology content areas. **It is to be understood that the above is offered only as an example and is not to be taken either as a suggestion or a prescription for developing curriculum.***

*In the following pages are the **CONTENT STANDARDS** for the Domains of Science and Technology by Grade-grouping.*

Content Standards

Inquiry Skills

Science and Human Affairs

Content Standards in the Physical Sciences
Properties of Matter

Grades K–4

- All objects occupy space and have mass. Objects have properties—such as size, shape, and mass—that may be compared and measured. Such properties can be used to describe, group, or classify the objects.
- Objects are made up of different kinds of materials. Materials have observable properties—such as color, texture, magnetic characteristics, solubility and different behaviors when heated or cooled. These properties can be compared and measured and are useful in describing, grouping, and classifying materials.
- Materials can exist in different states, including solid, liquid, and gaseous. Each state has different characteristic properties.
- Some properties of a material may change when it experiences some external change; other properties may not. The change in the material may be either physical, such as changes in state or appearance, or chemical, such as changes in composition.

Grades 5–8

- Different materials have characteristic properties that can be compared and measured. These properties allow materials to be distinguished from one another and often make them well suited to specific purposes.
- There are groups of elements that have similar properties, including very reactive metals, less-reactive metals, very reactive nonmetals (such as chlorine, fluorine, and oxygen), and some almost completely non reactive gases (such as helium and neon).
- A chemical change involves the transformation of one or more substances into new substances with different characteristic properties.
- The total mass of materials involved in any observed change remains the same. For example, mass is conserved during a change in state (such as solid to liquid or liquid to gas), or a chemical reaction.
- A change in either the pressure, temperature, or volume of a gas sample results in measurable, predictable changes in either of the other two properties.

Grades 9 –10

- Atoms of the same element have the same number of protons and electrons but may differ in the number of neutrons (isotopes).
- The observed properties of elements, each of which is made from a single type of atom, result from the number and arrangement of electrons in their atoms.
- Compounds form when atoms of two or more elements bond. Chemical bonds form when atoms share or transfer electrons.....
- Matter is made up of elements, compounds, and numerous mixtures of these two kinds of substances.
- Elements and compounds can be grouped into classes, based on similarities in their structures, and resulting properties....
- Radiation emitted during nuclear changes can affect living materials, (i.e., it can damage cells)....

Grades 11 - 12

- A useful model of the atom consists of a positively-charged nucleus (composed of protons and neutrons), surrounded by one or more negatively-charged electrons, held together by electrical forces described by Coulomb's Law.
- The properties of a compound can often be predicted from the structure of its smallest units (either molecules or ionic crystals).

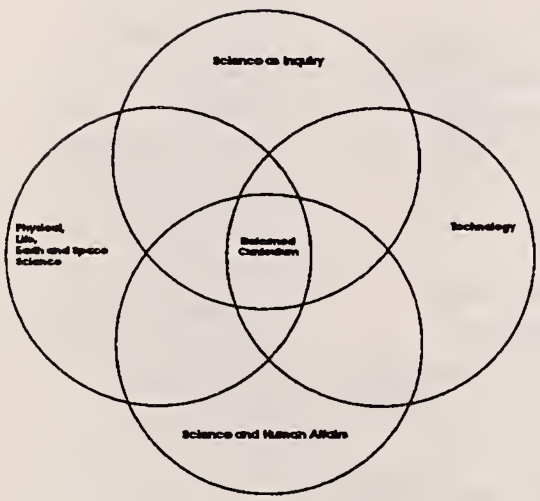
Technology

Connections with Mathematics

Diagram 5

This diagram traces the development, from kindergarten through twelfth grade, of a student's understanding of concepts related to "matter".

Kindergarten - 4th Grade Grouping Content Standards



Inquiry

Students should be able to:

- make observations
- ask questions about the natural world
- plan and conduct a simple investigation
- employ simple equipment to gather data
- recognize similarities, differences, and pat terns in data
- use data to construct a reasonable explanation
- communicate investigations and explanations

Human Affairs

Students should be able to:

- understand that many of today's technologies were not part of the world of their parents or grandparents
- understand some ways in which science and technology have improved our lives
- recognize that science and technology have also created problems we need to solve
- recognize that the decisions we make as individu-als have effects on others people
- understand that we (as individuals, groups and communities) can make decisions that change the natural environment

Physical Sciences

Physical Science

The physical sciences involve the study of energy and matter. In this framework, the physical sciences include both physics and chemistry. When studying the physical sciences students explore the composition, structure, properties, and reactions of matter as well as the relationships that exist between matter and energy. The developmental progression of students' studies in the physical sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades K - 4

Grades K-4 -- Young children acquire information about their world by examining, exploring, and manipulating common objects and materials in their environment. Their natural curiosity leads them to take advantage of many daily opportunities to compare, contrast, and describe common objects around them. Science classes in elementary schools should give children opportunities to continue these explorations in extended and more focused settings, using all their senses, and employing simple tools, such as magnifiers and measuring devices.

The study of the physical sciences in grades K-4 builds on the need for students to understand the characteristics of the objects and materials they encounter daily. At this age, children living in Massachusetts will have encountered snow, ice and water. They will know about different kinds of food and different building materials, such as wood, brick and steel. With a little help, they will begin to see that it is useful to classify objects and materials in different ways. Depending on the particular property chosen (color, hardness, etc.) things can be classified in different ways, resulting in different categories. Observing, manipulating, and classifying common objects leads children to reflect on similarities and differences among the materials that compose different objects.

Manipulations of objects by pushing, pulling, throwing, dropping, and rolling them can also lead students to think about the ideas such as movement, types of motion, how such motion can be usefully described, and — to a limited extent — how various kinds of motion can be controlled. Observation and manipulation leads children to reflect on similarities and differences in how things move or can be put into motion. In the early grades, simple sketches and single-word descriptions will provide a record of their observations. Over time, students should be encouraged to produce increasingly detailed drawings and richer verbal descriptions.

Physical Science

Experiences with light, heat, electricity, and the motion of objects can be used to create an interest in and an intuitive understanding of these phenomena and contribute to introduction and development of the concepts of energy and forces later.

Physical Sciences Content Standards
Grades K-4

Properties of Matter

- All objects occupy space and have mass. Objects have properties —such as size, shape, and mass —that may be compared and measured. Such properties can be used to describe, group, or classify the objects.
- Objects are made up of different kinds of materials. Materials have observable properties — such as color, texture, magnetic characteristics, solubility and different behaviors when heated or cooled. These properties can be compared and measured. These properties are useful in describing, grouping, and classifying materials.
- Materials can exist in different states, including solid, liquid, and gaseous. Each state has different characteristic properties.
- Some properties of a material may change when it experiences some external change; other properties may not. The change in the material may be either physical, such as changes in state or appearance, or chemical, such as changes in composition.

Position and Motion of Objects

- Motion of an object can be described as change in position with time, and can be represented on grids or graphs. The varieties of such motion include straight line, zigzag, or curving.
- An object's motion can be changed through the action of a push or pull on the object.
- Sound can be produced by an object that vibrates. Properties of sound such as pitch and loudness can be altered by changing the properties of its source or by putting the source into motion.

Forms of Energy: Light, Heat, Electricity and Magnetism

- The Sun supplies heat and light to the Earth.
- When placed in a beam of light, some objects cast shadows, while other objects bend or transmit the light.
- Things that give off light may also give off heat. Heat is produced by mechanical and electrical machines, and friction.

Grades K - 4

Physical Science

- When warmer things are put with cooler ones, the warm ones lose heat and the cool ones gain it until they are all at the same temperature.
- Some materials conduct heat much better than others.
- Without touching them, a magnet pulls on all things made of iron and either pushes or pulls on other magnets.
- Without touching them, material that has been electrically charged pulls on all other materials and may either push or pull other charged materials.
- Light, sound, heat and sparks can all be produced in electrical circuits using batteries as an energy source.

Vignette to be placed.

Grades K - 4

Life Science

Life Sciences

Life sciences involve the study of living organisms, of life processes and of the different levels of organization from the cell to the biosphere. They include the study of interactions between organisms and between organisms and their environments. Life sciences also address the unity and continuity of life as well as change and diversity. The study of life sciences enhances understanding of human life: how the human body works, develops and changes, and how humans interact with the environment. The developmental progression of students' studies in the life sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades K-4-- During the early years, children learn about the ways in which organisms maintain life, and the ways in which they interact with and depend on other living organisms and the nonliving parts of the environments in which they live. Through experience and observation, young students focus on the diversity and variability of familiar organisms. As they build understandings through direct experience with the living world, they are introduced to the scientific way of knowing. Young children are naturally curious and continuously engaged by interacting with the living world and trying to understand it.

To encourage a scientific way of knowing, teachers need to arrange and support opportunities for: careful observation of the plants and animals in the child's world, particularly those that can be studied directly in the children's schools, homes and immediate environments; and focused reflection on those observations in order to allow children to make meaning of their experience. For children in the early grades, scientific observations are those that have been made carefully, using their senses and simple magnifiers and measuring tools, recorded using words or pictures, and communicated to their classmates. At this stage of children's development, the use of scientific terms is not critical.

Finding patterns in nature is a shared activity. This may take time and may involve revisiting what was observed, as children's observations are often strongly influenced by their beliefs and prior experiences.

Grades K - 4

Life Sciences Content Standards Grades K-4

Characteristics of Organisms

- Plants and animals are composed of different parts, serving different purposes and contributing to the well-being of the whole organism.
- Many different kinds of plants and animals live on Earth today. Living organisms can be classified into species on the basis of similarity in appearance and behavior. Within each species, individuals vary.
- Plants and animals are alive and go through predictable life cycles. These cycles differ from species to species, but all include growth, development, reproduction and death.

Diversity and Adaptation of Organisms

- Different plants and animals have external features that help them thrive in different kinds of places.
- Some kinds of organisms that once lived on earth have completely disappeared, although they were something like others that are alive today.
- Individuals of the same kind differ in their characteristics, and sometimes the differences give individuals an advantage in surviving and reproducing.
- Fossils can be compared to one another and to living organisms according to their similarities and differences. Some organisms that lived long ago are similar to existing organisms, but some are quite different.

Heredity

- Traits and characteristics are passed from parents to offspring.
- There are variations as well as similarities among individuals of the same species.
- Offspring resemble but are not exactly like their parents.
- Some likenesses between children and their parents, such as eye color in human beings or fruit or flower color in plants, are inherited.

Organisms and Environments

- Most plants and animals need air, food water, light and suitable environments. Green plants make their own food. Animals consume plants or other animals for food.

Life Science

- Decomposers break down dead plants and animals for food.
- Living organisms meet their needs by interacting with living and nonliving parts of environments in which they live. Plant and animal species are dependent on each other to maintain life. Each species has features that enable it to live and reproduce in a particular environment (habitat).
- Natural forces affect the environments in which plants and animals live. Natural forces affect individual organisms, as well as groups of organisms.

Vignette to be placed.

Grades K - 4

Earth and Space Sciences

Earth and Space Science

In earth and space sciences, students investigate the origin, structure, and physical phenomena of the earth and study the earth as a part of the universe. Earth and space sciences include areas such as geology, meteorology, oceanography, and astronomy. The study of earth and space sciences deepens our understanding of processes such as plate tectonics, properties of geological materials, changes in the earth's topography over time, and the place of the earth in the universe. The discussion below has been adapted from the NRC's *National Science Education Standards: Fundamental Understandings for Earth and Space Sciences* (August 1993).

Grades K-4 -- The curiosity of young children leads them to take advantage of many daily opportunities to compare, contrast, and describe the materials of which the earth is made, and the visible objects in the night sky. Young children bring these experiences to school and are given opportunities to continue these explorations in extended and more focused settings, using all their senses and simple tools, such as magnifiers, measuring devices and telescopes.

The study of the earth and space sciences in Grades K-4 builds on the need for students to understand important characteristics of materials they encounter daily —such as soil, rocks, water and air— and to begin seeing the utility of classifying them into various categories, based on their properties. In addition, observing objects such as the moon, sun, and stars leads students to think about the characteristic movement of these objects and how their motion can be described.

In Grades K-4 students' understanding of properties and changes of earth's materials, and of objects in the sky should not be cultivated through isolated events or activities. Rather, these ideas should develop within broader explorations of the child's world. Investigations of organisms (see Life Sciences) or of other phenomena (Physical Sciences) can make important contributions to children's basic ideas about the materials of which our earth is made and the nature of the movements of the earth and other planets.

Grades K - 4

Earth and Space Science

Earth and Space Sciences Content Standards
Grades K-4

Properties and Changes of Earth's Materials

- Earth's surface is composed of rocks and soils, water, and living organisms.
- Water can be a liquid, a solid, or a vapor.
- Some events in nature have a repeating pattern. The weather changes some from day to day, but things such as temperature and precipitation show the same monthly rhythms from year to year.
- Air has properties that can be sensed and measured, such as wind speed and direction, temperature, the occurrence of clouds, and the fall of precipitation. The ensemble of these properties and events for particular place and time is called the weather.
- Rocks are made of minerals. Different types of rock have different physical properties, such as hardness, color, shape, and texture.
- Rocks come in many sizes and shapes, from boulders to grains of sand and even smaller.
- Change is something that happens to earth materials.
- Water flows downhill in streams and rivers, or accumulates in lakes and puddles.
- Fossils provide evidence of Earth's history, and show how plants, animals and environments have changed over time.

Objects in the Sky

- The earth is one of several planets that orbit the sun, and the Moon orbits around the earth.
- The Sun, Moon, planets, meteors, clouds and other objects in the sky can be identified by properties such as size, shape, color, brightness, and movement.
- The Sun provides light and heat.
- The Sun can be seen only in the daytime, but the Moon can be seen sometimes at night and sometimes during the day. Because the earth rotates, the Sun, Moon, and stars all appear to move slowly across the sky.
- The Moon looks a little different every day, but looks the same again about every four weeks.

Vignette to be placed.

Grades K - 4

Technology

Investigating the salinity of the water in Vineyard Sound is a pursuit of science. Creating a way to make this salt water drinkable is a pursuit of technology. Simply stated, science is an attempt to understand the natural world; technology is an attempt to create solutions to human problems. Yet science and technology are not necessarily mutually exclusive fields of endeavor. In fact, they are intimately bound together. Scientific understanding provides us with a basis for recognizing, and proposing solutions for, problems of human adaptation. Technology enables us to further our knowledge about the world. The interrelationships between science and technology — and their connections to educational goals — are depicted in the diagram.

People are surrounded by products of the technological design process. As pointed out in the AAAS *Benchmarks*,

> Young children are veteran technology users by the time they enter school. They ride in automobiles, use household utilities, operate wagons and bikes, use garden tools, help with the cooking, operate the television set, and so on. Children are also natural explorers and inventors, and they like to make things. (p. 44)

It is important to capitalize upon these experiences and natural inclinations. Learners should have opportunities to identify problems of human adaptation to the environment and then, to apply problem-solving strategies, design investigations, and build models and physical tools in solving these problems. The content area "Technological Design" emphasizes the importance of creating solutions through the process of design. The design process allows students to identify or discover a problem, apply scientific knowledge and skills in conceptualizing a solution, weigh options, make decisions, construct and test tools, reflect on results, and reformulate their ideas.

Some students are excellent at doing things with their hands but have difficulty verbally representing what they are doing; others have strong verbal skills. The school environment tends to favor students of the latter type. Because design problems involve the construction of prototypes or structures that embody formal ideas, they can stimulate students to transfer knowledge between the two realms.

Technology

Technology Content Standards
Grades K-4

The nature and impact of Technology

- technology is used to satisfy human needs through communication of its messages, construction of shelter, manufacture of necessities, transportation of people and goods, generation of power, and advancement of the growth of bio-related technologies.
- daily activities involve the use of technology.
- all technologies have positive and negative impacts on people and the environment.

Technology yesterday, today and tomorrow

- the history of technological change and technological developments has significantly accelerated human progress, such as the development of writing, to desk to publishing and the hot air balloon to the space shuttle.
- technological inventions an innovations have been developed by males and females from various racial and cultural backgrounds including individuals from Massachusetts.
- early tools and devices have contributed to human progress.

The tools and machines of Technology

- tools and machines are used to process materials, energy, information as well as to model solutions to design problems.
- tools and machines extend human capabilities.

Resources of Technology

- various material resources have different properties and characteristics and some are better than others for a specific use.
- a variety of materials such as wood metal, plastic, fabric and clay can be used to make simple products.
- some materials can be recycled while others cannot.
- the resources of technology (people, information, tools and machines, materials, energy, capital, time) are necessary in any technology endeavor.

Grades K - 4

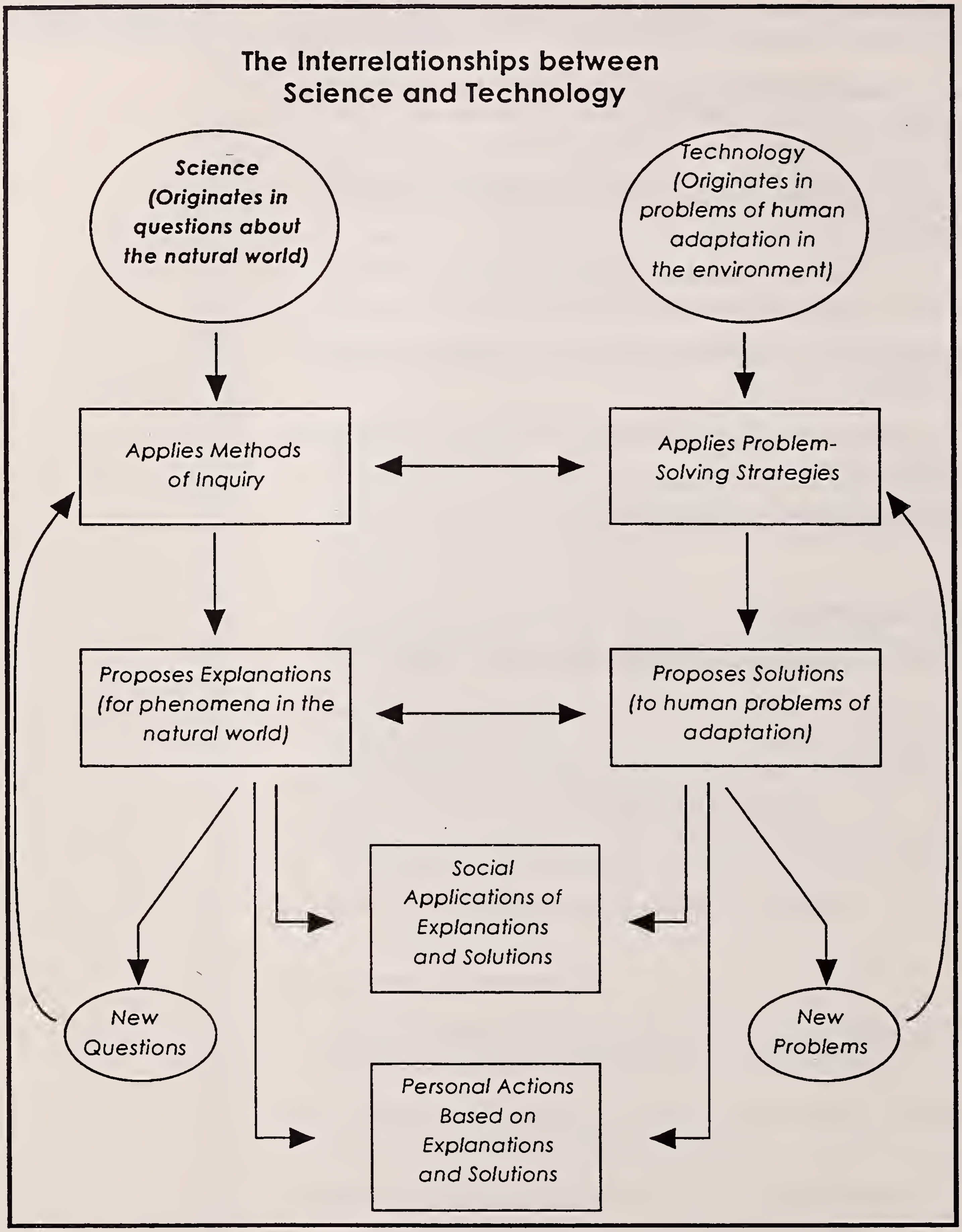
Technological areas of communication, construction, manufacturing, transportation, power and bio-related technologies

- messages are communicated graphically and electronically using drawing, computer hardware and software and simple electronic devices.
- many types of structures exist (ie: residences, skyscrapers, bridges, tunnels, airports).
- artifacts are manufactured from materials such as wood, plastic, paper, metal and clay using cutting and shaping tools.
- people and goods are transported using boats, automobiles, trucks, airplanes and space vehicles.
- energy is converted into different forms, mechanical linkages and gears transmit power.
- food products are produced and preserved.

Technology in a global society

- major commercial producers and distributors directly affect our lives.
- many companies that have innovative products and/or services, including companies located in Massachusetts or New England.

Vignette to be placed.



Grades K - 4

Students should be able to:

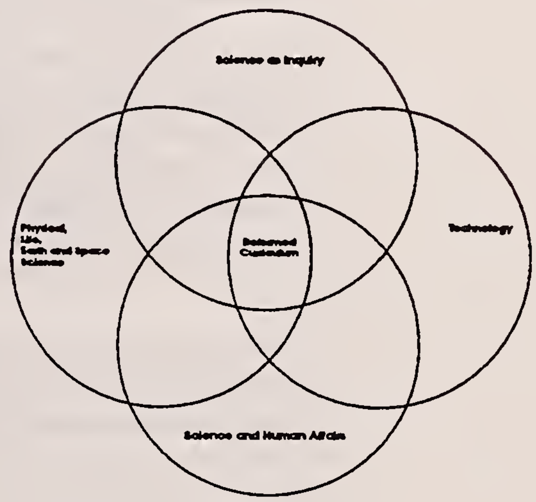
- State a problem
- Propose a solution
- Implement a solution
- Evaluate the results

5th - 8th Grade Grouping Content Standards

Inquiry

Students should be able to:

- make observations
- ask questions for scientific investigations
- design and conduct a scientific investigation
- use appropriate tools and techniques to gather data
- construct explanations and models using evidence
- think critically and logically about the relationships between evidence and explanations
- recognize and analyze alternative explanations and procedures
- communicate scientific procedures and explanations



Human Affairs

Students should be able to:

- understand that the decisions we make as individuals, groups and communities can affect society and the natural environment, and that these changes are not always easy to reverse
- recognize that while technology can help us to manage societal and environmental problems, it can also have a negative impact on society and on the natural world
- understand how science and society have influence each other in the past
- understand the influences science and technology have on today's society
- develop skills in applying scientific knowledge to making decisions about societal problems and recognize that using the skills responsibly is an essential part of being a citizen in today's world

Science Vignette: A passion for students, teaching, and the sea

"There are more people alive today than have ever died." I wrote the sentence on the board that I had heard Robert Ballard say last night. Dr. Ballard, the Woods Hole scientist who found the Titanic, the Lusitania, and the Bismarck and is pioneering robotics in undersea exploration, was the speaker at a dinner meeting I attended. My eighth grade students came into the class, sat down, read the statement on the board, and immediately began talking among themselves. After about five minutes of active dialogue, one student asked, "What are we supposed to do today?" "You're doing it," I replied. "You're reacting to this statement the same way I reacted. What are the implications for us?"

I teach science, or more specifically, ocean sciences, to the 125 eighth grade students enrolled in a middle school in Cape Cod. The inception of the program, fifteen years ago, was a combination of a love I have for the ocean, and the frustration I felt for the general science program I was teaching. I believed I could teach the same science better and make it more relevant using local applications. I scrapped a botany program that was based on corn, and a zoology program on perch dissection. I convinced the principal to let me try out the ocean centered approach.

My classroom was equipped with two microscopes, and I needed 25! The principal agreed to buy two a year, but finally upped the amount to six per year. I begged and borrowed other equipment, sending a wish list to parents, the local museums, and the newspaper want-ads. In response, the class received fish tanks, air pumps, general aquarium supplies, and five refrigerators. The local museum of natural history gave me a catalog of free resources, and I spent evenings writing about 75 letters requesting curriculum materials.

My major focus was on getting the students involved doing science. I found a fantastic book, Experiments in Oceanography, that had suggestions for hands-on activities plus how to build the needed tools, for example, how to build a plankton net out of panty-hose.

Throughout the years the course has evolved to a routine schedule, but one that I hope is engaging and challenging for the students. "Creature Feature" happens every Monday. I bring in 10-15 specimens of a creature, perhaps star fish or horseshoe crabs, and students are to look at the creature, and say as much as they can about what they observe. At first this is difficult for the students. They say things like it has claws or a shell. But the more they do this, the better they become in identifying distinguishing characteristics of animals.

On Fridays we do experiments in physical oceanography, examining nonliving aspects of the ocean, such as waves, tides, and salinity. One way we have investigated currents is by repeating Benjamin Franklin's experiment and inserting cards in bottles and dropping them into the ocean. For more than ten years, on March 20, the Coast Guard has taken students 17 miles out to the same buoy in shipping lanes. Of the 2000 bottles that have been set adrift, about 30 have been returned from such diverse locations as Ireland, Scotland, Portugal, France, Sweden, Bermuda, Nova Scotia, and Ipswich.

Reading, writing, mathematics, and history are integrated into the course. Each spring students collect data on herring migration, when herring, in a way somewhat similar to salmon, return to their birthplace to lay eggs. (Unlike salmon, herring don't die at the end of this process.) Students record the

weight, length, and sex for a sample of herring, find the average for the data, and compare their findings with those recorded for previous years. One interesting finding has been that following the blizzard of 1978, the 1979 herring catch had an average weight drop and a length increase! Students conjecture why this might have occurred since that during the blizzard the herring were many miles south of New England. Students also learn about one of the major commercial uses of herrings as fertilizer, and that Indians also used herrings for fertilizer and this saved the lives of many Pilgrims.

The course is rigorous and demanding for the students. While eighth graders vigorously protest structure, internally they want it. Students receive a monthly learning agenda that specifies all assignments, including homework. I tell them that they will get no surprise quizzes, no weekend homework, and in exchange, there are no excuses. Work must be done on time. For some assignments, particularly lab ones, students work collaboratively. Other assignments are done independently. My overall emphasis, however, is to treat this program as an attitude not a course—we are interrelated to the ocean, and the ocean profoundly affects our lives.

When we moved into a new middle school, it was decided that each grade level choose a name and a theme. I suggested "Stewardship"—building the sense of responsibility for the earth. A ship's wheel became the logo, with the spokes representing the different disciplines, helping to steer the students through the sea of life. Students learn, by engaging in a variety of activities in each subject, that stewardship centers around being proactive, rather than reactive or inactive. For example, eighth graders help clean up garbage on the beach on Saturdays, not because they left it there or simply to help make the beach more attractive, but because of the belief that even though this is not our garbage, we are all going to be poisoned by it.

I want to instill in my students that each one of them can make a difference. In 1976 one of my students wrote a letter to our congressman, that was recorded in the congressional letter, stating how precious Stellwagen Bank, an underwater shelf used as a habitat by local whale populations, is and recommending that it be declared a sanctuary. In 1992 that actually happened. My former student, Richard Comeau, is now an officer in the merchant marine.

All human beings need a passion in their lives. I have found mine. When my students see there is a purpose to what they do in school, when they make connections between school and the world, their eyes light up, and I hope that maybe, they will get a sense of the excitement that can develop when one is committed to making a difference in events that affect the quality of all our lives.

George Kurlychek, Harwich Middle School

Physical Science

Grades 5 - 8

Physical Sciences

The physical sciences involve the study of energy and matter. In this framework, the physical sciences include both physics and chemistry. When studying the physical sciences students explore the composition, structure, properties, and reactions of matter as well as the relationships that exist between matter and energy. The developmental progression of students' studies in the physical sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades 5-8-- As students move through grades 5-8, they should gain a more sophisticated idea of what scientific investigation entails–from the initial "guesses" or hypotheses, through the gathering of data, comparison of the results with the hypotheses, and the formulation of higher-level conclusions. Their experiments will become more quantitative as they learn to make accurate measurements using a variety of instruments. Through this process, their attention will shift from properties of objects, to characteristics properties of substances, to the question of what model of matter can be created to explain those properties.

The introduction of the particulate model presents a major and exciting challenge for both teachers and students as they move toward the end of the grade 5-8 period. Most students will have heard of atoms and have some primitive appreciation of the idea that what we see is composed of tiny particles, but they will have almost no understanding of the evidence for the particulate model or the logical arguments that led to the development of one model in preference to others. While the full line of evidence and argument for atoms and subatomic particles cannot be introduced at this level, discussions of how particular experimental results contributed to the particulate model can provide a rich illustration of one of the greatest empirical adventures of the 20th century — the persistent efforts of scientists to unlock the secrets of the atom.

The energy strand continues from the lower grade levels with an increased emphasis on quantitative descriptions, and with the introduction of some formal terminology scientists use in their field. The idea of energy and its inter-conversions begins to emerge towards the end of these grades as students observe and measure relationships among light, heat, sound and electricity. The study of motion and forces leads to a more comprehensive understanding of energy, and provides the basis for further learning at the grade 9-12 level.

Physical Science

Grades 5 - 8

Properties of Matter

- Different materials have characteristic properties that can be compared and measured. These properties allow materials to be distinguished from one another and often make them well suited to specific purposes.
- There are groups of elements that have similar properties, including very reactive metals, less-reactive metals, very reactive non-metals (such as chlorine, fluorine, and oxygen), and some almost completely non reactive gases (such as helium and neon).
- A chemical change involves the transformation of one or more substances into new substances with different characteristic properties.
- The total mass of materials involved in any observed change remains the same. For example, mass is conserved during a change in state (such as solid to liquid or liquid to gas), or a chemical reaction.
- A change in either the pressure, temperature, or volume of a gas sample results in measurable, predictable changes in either of the other two properties.

Particulate Model of Matter

- It is possible to devise a particulate model for matter that accounts for the observed properties of substances.
- Experimental evidence supports the idea that matter can be viewed as composed of very small particles, (atoms and molecules) which are in constant motion. Particles in solids are close together and not moved about easily. Particles in liquids are not as close together and move about more easily. Particles in gases are quite far apart and move about freely.
- The conservation of mass is consistent with the particulate model that describes changes in substances as the rearrangement of the component particles. Since the number of these particles remains the same, the total mass of the sample remains unchanged.

Motions and Changes in Motion

- Several forces acting on objects can be regarded as pushes or pulls that can either reinforce or cancel each other. Forces are quantities that have both magnitude and direction.
- All forces have magnitude and direction. Forces acting in the same direction reinforce each other. Forces acting in different directions may detract or cancel each other.
- An object's motion can be described and represented graphically in terms of direction, speed, velocity, and [or] position versus time.

Transformations of Energy

Physical Science

- Energy can not be created or destroyed but exists in different interchangeable forms, such as light, thermal, electrical, mechanical, vibrational.
- Temperature changes in a sample of matter are related to the loss or gain of thermal energy by that sample. Temperature changes usually cause changes in solubility.
- Energy comes to the Earth from the Sun, both as visible light and as invisible electromagnetic radiation, such as ultraviolet, infrared and radio waves. The amount of each type of radiation reaching the Earth depends on the absorption properties of the atmosphere.
- Light, which has color, brightness, and direction associated with it, can be absorbed, scattered, reflected or transmitted by intervening matter. Refraction is the process by which light's direction can be changed by passing from one medium to another.
- Energy changes involved in physical or chemical changes can be measured in different forms such as heat, light, and sound.
- The principles of electrical circuits can be demonstrated using wires, batteries and bulbs to analyze electrical energy resistance, current and power. Electric currents can also be used to produce electromagnetic coils of wire, and, conversely, a moving magnet can generate a current in a circuit.

Grades 5 - 8

Life Sciences

Life Science

Life sciences involve the study of living organisms, of life processes and of the different levels of organization from the cell to the biosphere. They include the study of interactions between organisms and between organisms and their environments. Life sciences also address the unity and continuity of life as well as change and diversity. The study of life sciences enhances understanding of human life: how the human body works, develops and changes, and how humans interact with the environment. The developmental progression of students' studies in the life sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades 5-8-- The basic understandings of the K-4 years are elaborated and become more sophisticated in grades 5-8 as students study a range of organisms, engaging in experimentation and field study. Students at this age also have the fine motor skills to work with a light microscope and can develop the ability to accurately interpret what they see. The microscope allows students to see the cell and to build an understanding of the cell as the basic building block of life. A central focus of study becomes life processes and the relationship of the life processes of cells with those of organisms. A related theme is that of living systems. Students explore cells as subsystems of multicellular organisms. With a microscope, they can also be introduced to the world of single-celled organisms. Over time, they come to understand that single-celled and multicellular organisms are both living systems composed of interacting subsystems.

As the range of their experience expands and they learn to analyze data and think abstractly, students will look at variation and diversity in more sophisticated ways. The systems they use for the classification of organisms will become more complex and will start to conform to the classification systems used within the scientific community. With greater experience, more refined observational strategies and tools, and a greater understanding of reproduction, students will begin to build an understanding of heredity, the continuity of life and, eventually, of evolution.

At this level students begin the study of ecosystems, that is, environments that are conceptually bounded for the purpose of studying the living things within them. This adds depth to their under-

Grades 5 - 8

standing of how living things interact with each other and with nonliving parts of the system. Studies of ecosystems support the development of an understanding of the interdependence of all organisms, and this, in turn, provides the basis on which they can begin to study evolution.

Life Sciences Content Standards
Grades 5-8

Characteristics of Organisms

- Plants, animals, fungi, and various types of microorganisms are major categories of living organisms. Each category includes many different species.
- Cells are the basic units of life. They are the smallest unit of life that can reproduce themselves. Organisms may be single or multicellular.
- In single cells there are common features that all cells have as well as differences that determine their function.
- Cell replication results not only in the multiplication of individual cells, but also in the growth and repair of multicellular organisms.
- All organisms, whether single of multicellular, exhibit the same life processes, including growth, reproduction and the exchange of materials and energy with their environments.
- In multicellular organisms, cells can differ in many ways, assuming different appearances and carrying out specialized functions
- Complex multicellular organisms are interacting systems of cells, tissues or organs that fulfill life processes through mechanical and chemical means, including procuring or manufacturing food, and breathing and respiration.

Diversity and Adaptation of Organisms

- Short-term changes in available food, moisture, or temperature of an ecosystem may result in a change in the number of organisms in a population or in the average size of individual organisms. Long term changes may result in the elimination of a population.
- In both the short and long term (millions of years), changes in the environment have resulted in qualitative and quantitative changes in the species of plants and animals that inhabit the Earth.

Life Science

Heredity, Reproduction and Development

- Reproduction is the key characteristic of living organisms. In some instances, a single organism is involved, such as when a single yeast cell divides to form two cells (asexual). In other instances, two parents are involved, with one parent producing eggs, and the other parent producing sperm. When the egg and the sperm unite, a new organism forms (sexual).
- Differences passed on to a new organism are the result of changes in genetic material in the parent organism's reproductive cells. Organisms that have two parents receive a full set of genetic instructions, specifying individual traits from each parent. Offspring exhibit traits from each parent.
- Sorting and recombining of the genetic material of parents during reproduction produce the potential for variation among offspring.
- There are minor differences among individuals from the same population or among individuals of the same species. Some differences are acquired by the individual and affect only that individual, while other differences can be passed on to the individual's offspring.

Ecosystems and Organisms

- Species depend on one another. Interactions of organism with each other and nonliving parts of their environments result in the flow of energy and matter throughout the system.
- Energy is supplied to an ecosystem primarily via sunlight. Plants convert light energy into stored energy which the plant, in turn uses to carry out its life processes. This serves as the beginning of the food chain for all animals.
- Plants, animals, fungi, and microorganisms interact in a variety of ways. Through these interactions matter is cycled and recycled, and energy flows through ecosystems.
- Organisms can be classified according to the function they serve in a food chain (any single organism can serve each of these functions): production of food, consumption of food, or decomposition of organic matter.

Vignette to be placed.

Grades 5 - 8

Earth and Space Science

Earth and Space Sciences

In earth and space sciences, students investigate the origin, structure, and physical phenomena of the earth and study the earth as a part of the universe. Earth and space sciences include areas such as geology, meteorology, oceanography, and astronomy. The study of earth and space sciences deepens our understanding of processes such as plate tectonics, properties of geological materials, changes in the earth's topography over time, and the place of the earth in the universe. The discussion below has been adapted from the NRC's *National Science Education Standards: Fundamental Understandings for Earth and Space Sciences* (August 1993).

Grades 5-8-- In the early part of grades 5-8, students continue to investigate the properties of geological materials and the substances of which they are composed. In the process, their experiments will become more quantitative as they expand their ability to make measurements, design and conduct investigations and come to richer understandings. Attention shifts from the properties of particular objects towards an understanding of the place of the earth in our solar system and changes in the topography and composition of the earth's crust through time.

Earth and Space Sciences Content Standards
Grades 5-8

Interactions and Cycles in the Earth System

- The Earth is mostly rock. Three-fourths of its surface is covered by a relatively thin layer of water (some of it frozen), and the entire planet is surrounded by a relatively thin blanket of air. It is the only body in the solar system that appears able to support life.
- Everything on or anywhere near the Earth is pulled toward the earth's center by gravitational force.
- Earth's surface consists of an array of moving crustal plates. Volcanoes form where one plate slips under another or where two plates spread apart. Earthquakes and mountain building occur wherever plates interact.
- Landforms of various shapes and sizes result from both constructive and destructive processes. Volcanic eruptions, sediment deposition, tectonic uplift, and other processes serve to build up landforms, and weathering and erosion wear them down.

Grades 5 - 8

Earth and Space Science

- The interior of the Earth is hot. Heat flow and movement of material within the earth cause earthquakes and volcanic eruptions and create mountains and ocean basins. Rocks are classified as sedimentary, igneous, and metamorphic by their formation.
- Rocks are continuously being modified by processes such as weathering, erosion, deposition, compaction, cementation, melting, heating (without melting), pressure, and crystallization. Sequences of these processes, collectively referred to as the rock cycle, occur continuously as materials move on or through earth's upper crust.
- Soil is formed by the weathering of rock and the decomposition of dead plants and animal debris. Soil is essential for the survival of most life on land, and is the connection between many of the living and nonliving constituents of the Earth System.
- The Moon's orbit around the earth once in about 28 days changes what part of the moon is lighted by the Sun and how much of that part can be seen from the Earth—the phases of the Moon.
- Water in the Earth System exists naturally in all three states. Driven by gravity and heat, water continuously circulates through the Earth's crust, oceans, and air in the water (hydrologic) cycle. Water plays important roles in regulating earth's climate.
- Because the Earth turns daily on an axis that is tilted relative to the plane of the Earth's yearly orbit around the Sun, sunlight falls more intensely on different parts of the earth during the year. The difference in heating of the Earth's surface produces the planet's seasons and weather patterns.
- Earth's oceans are a reservoir of nutrients, minerals, dissolved gases, and the major source of water vapor for the atmosphere. The storage of heat in ocean waters and its poleward transport by ocean currents greatly affect Earth's climate. Important elements of Earth's biota are contained within the oceans.
- Properties of the atmosphere comprising the weather, such as pressure, temperature, humidity, wind speed and direction, precipitation, and amount and type of clouds, occur in seasonally-varying sequences with a period of a few days. When the weather at many locations is considered, these sequences are seen to represent the passage of three-dimensional patterns of pressure, temperature and the other quantities.
- Global wind patterns within the atmosphere are determined by the unequal heating between the equator and poles, Earth's rotation, and the distribution of land and ocean. Consequently, weather in northern and southern mid-latitudes tends to move

Grades 5 - 8

eastward while in the tropics it moves westward. Atmospheric winds transport heat poleward from the warm tropics, helping to maintain Earth's climate.

- Clouds form from the condensation of water vapor to form either droplets or crystals. Some types of clouds may produce rain, snow or other forms of precipitation. Clouds reflect much of the sunlight intercepted by Earth, while at the same time returning to Earth's surface a large fraction of the far infrared energy emitted from the surface. These two effects are important elements in determining Earth's global climate.
- Climate is the long term average of weather. Local climate changes over periods of years or decades, while global climate changes much more slowly. Climate changes over Earth's history have profoundly affected the evolution of life forms.
- The atmosphere and the oceans have a limited capacity to absorb wastes and recycle materials naturally.
- Human activities, such as reducing the amount of forest cover, increasing the amount and variety of chemicals released into the atmosphere, and intensive farming, have changed the Earth's land, oceans, and atmosphere

Earth's History

- The positions of Earth's continents have changed over millions of years, as they ride upon moving crustal plates. The changing configuration of continental land masses affect their major physical features, currents in the oceans, and local and global climates.
- Rocks record events of Earth's history, documenting plate movements, volcanic eruptions, cycles of erosion and deposition, and the evolution of life. Ice cores from the polar regions and from glaciers, sediments from lakes and ocean floor, and tree rings document more recent events, such as the advance and retreat of the last great ice sheets, "Little Ice" ages, and droughts and floods.
- Fossils are evidence of past life. The types, numbers and distributions of fossils provides information about how life and environmental conditions have changed over time.

Earth in the Solar System

- The patterns of stars in the sky stay the same, although they appear to move across the sky nightly, and different stars can be seen in different seasons.
- Telescopes magnify the appearance of some distant objects in the sky, including the Moon and the planets. The number of stars that can be seen through telescopes is dramatically greater than can be seen by the unaided eye.

Earth and Space Science

- Planets change their positions against the background of stars.
- The Solar System contains the central Sun, and known planets, their moons, and many asteroids, meteoroids and comets orbit the Sun. The planets differ in size, temperature, composition, surface features, and number of rings and moons.
- The Sun is a medium-sized star located near the edge of a disk-shaped galaxy of stars, part of which can be seen as a glowing band of light that spans the sky on a very clear night.
- The Sun is many thousands of times closer to the earth than any other star. Light from the Sun takes a few minutes to reach the Earth, but light from the next nearest star takes a few years to arrive.
- The universe contains many billions of galaxies, and each galaxy contains many billions of stars.
- Like all planets and stars, the Earth is approximately spherical in shape. The rotation of the Earth on its axis every 24 hours produces the night-and-day cycle. To people on Earth, this turning of the planet makes it seem as though the sun, moon, planets, and stars are orbiting the Earth once a day.
- Earth undergoes many motions as it moves through space. The planet is a spherical body that rotates on its axis, completing one rotation each day; this is the cause of day and night. Earth revolves around the Sun in a year of approximately 365 days.
- Earth has a large natural satellite, the Moon, that circles the planet approximately every 29 days. The motion of the Moon about earth and the location of the Sun relative to Earth and its Moon are responsible for the regularly occurring patterns of Moon phases and eclipses.
- Gravity is a force that produces an attraction between all forms of matter. Gravity holds objects on the spherical Earth, and acts across space to hold the Moon in its orbit around Earth and the planets in their orbits around the Sun. The gravitational effects of the Moon are responsible for the tides in Earth's oceans.
- The Sun produces energy and is the major source of heat and light for Earth. The energy received from the Sun as heat and light drive many processes on Earth's surface and in its atmosphere.

Vignette to be placed.

Grades 5 - 8

Technology

Technology

Investigating the salinity of the water in Vineyard Sound is a pursuit of science. Creating a way to make this salt water drinkable is a pursuit of technology. Simply stated, science is an attempt to understand the natural world; technology is an attempt to create solutions to human problems. Yet science and technology are not necessarily mutually exclusive fields of endeavor. In fact, they are intimately bound together. Scientific understanding provides us with a basis for recognizing, and proposing solutions for, problems of human adaptation. Technology enables us to further our knowledge about the world. The interrelationships between science and technology — and their connections to educational goals — are depicted in the diagram.

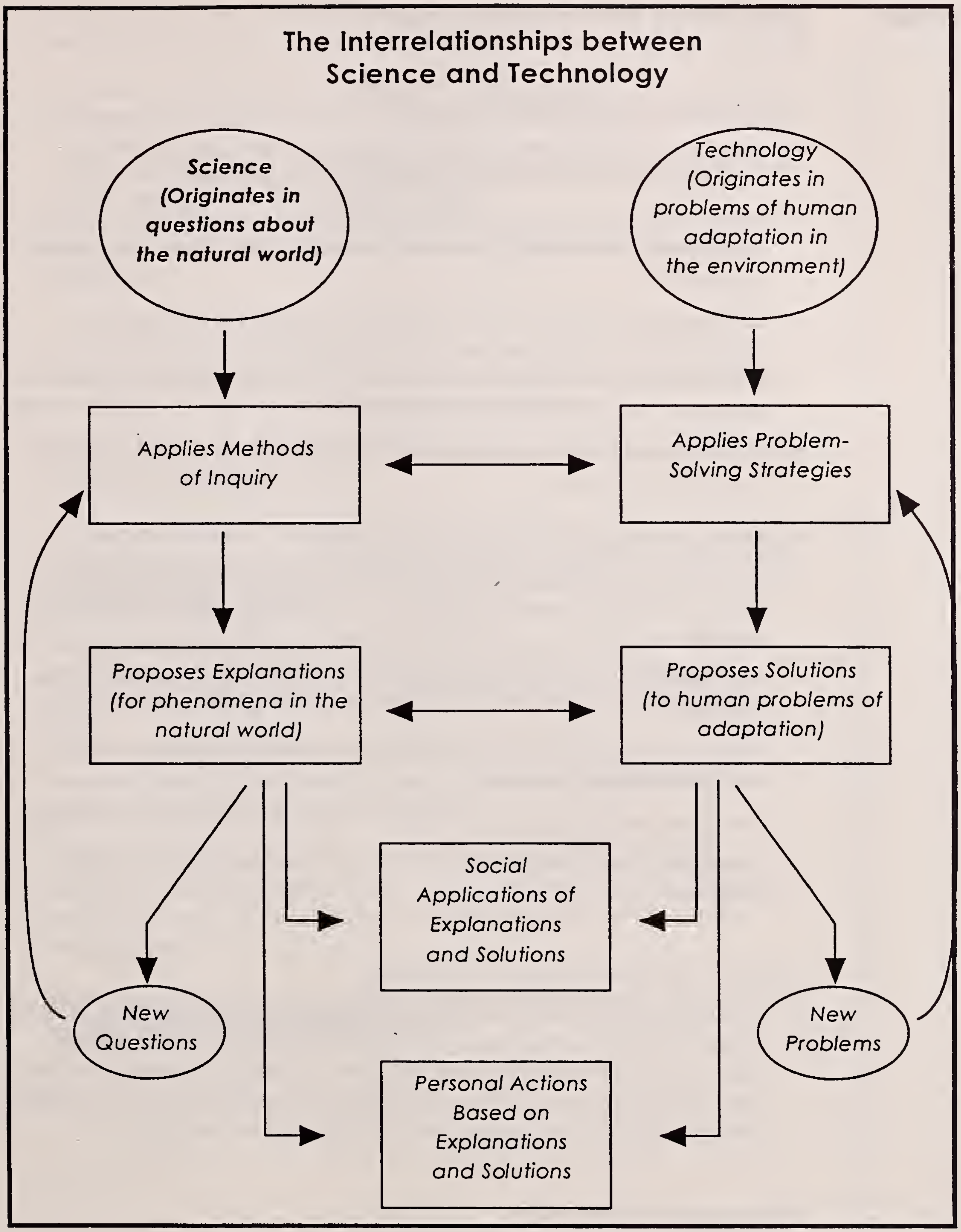
People are surrounded by products of the technological design process. As pointed out in the AAAS *Benchmarks*,

Young children are veteran technology users by the time they enter school. They ride in automobiles, use household utilities, operate wagons and bikes, use garden tools, help with the cooking, operate the television set, and so on. Children are also natural explorers and inventors, and they like to make things. Grades 5 - 8 Earth and Space

It is important to capitalize upon these experiences and natural inclinations. Learners should have opportunities to identify problems of human adaptation to the environment and then, to apply problem-solving strategies, design investigations, and build models and physical tools in solving these problems. The content area "Technological Design" emphasizes the importance of creating solutions through the process of design. The design process allows students to identify or discover a problem, apply scientific knowledge and skills in conceptualizing a solution, weigh options, make decisions, construct and test tools, reflect on results, and reformulate their ideas.

Some students are excellent at doing things with their hands but have difficulty verbally representing what they are doing; others have strong verbal skills. The school environment tends to favor students of the latter type. Because design problems involve the construction of prototypes or structures that embody formal ideas, they can stimulate students to transfer knowledge between the two realms.

Grades 5 - 8



Grades 5 - 8

Students should be able to:

- Identify a problem
- Design a solution - criteria/constraints
- Implement the solution
- Evaluate the solution

Technology

Technology Content Standards
Grades 5 - 8

The nature and impact of Technology

- people are influenced by technology and changes in living conditions as a result of technological development.
- technological progress has been the result of cumulative work over many centuries by men and women from various cultures and races.
- technological devices have improved the quality of life for individuals. The impacts of technology can be intended, unintended, desired, and/or undesired, helpful or destructive depending upon how people employ it.
- although technology solves many problems, new problems may result from its use.

Technology yesterday, today and tomorrow

- the evolution of technology led the change from an agriculture based to an industrially-based, to an informational-based society.
- technology is growing at a faster rate today than ever before in history.
- innovations and inventions satisfy human biological, physical and physiological needs.
- complex technological tools have evolved to their present state.
- emerging technologies create new jobs and make other jobs obsolete.

The tools and machines of Technology

- a range of tools and machines including measuring tools, hand tools, machines, electronic tools and instruments and optical tools are used to implement solution to design problems in the technological areas of communication, construction, manufacturing, transportation, power and bio-related technology.

Resources of Technology

- the choice of materials depends upon their properties and characteristics and how they interact with other materials.
- the result of material tests (ie: hardness, tensile strength, conductivity), suggest appropriate uses for materials.

Grades 5 - 8

- some materials are biodegradable and others are not. There are benefits to recycling materials into useful products.
- there are seven resource categories. These categories are: people, information, tools&machines, materials, energy, capital and time.
- resources on earth are limited and people must use these resources wisely and dispose of them responsibly.
- synthetic materials can help reduce the depletion of some scarce resources

Technological areas of communication, construction, manufacturing, transportation, power, and bio-realted technologies

- information is communicated graphically and electronically, reducing communication errors.
- a manufacturing enterprise produces a product by choosing the appropriate processes of casting, forming, separating, conditioning, assembling and finishing.
- the processes used in construction are: preparing the site for the structure, setting the foundation for the structure, erecting the structure, installing facilities when required and completing the site.
- transportation systems are devised to transport people and products on land, water, air and in space.
- power systems are used to convert and transmit mechanical, electrical, fluid and heat energy. There are limited (ie: fossil fuels), unlimited (ie: solar, gravitational) and renewable (ie: biomass) energy sources.
- bio-related processes are used in manufacturing and agriculture (ie. hydroponics and water purification).

Technology

Technology in a global society

- technological factors affect the standard of living in various nations.
- the application of technology has led to the growth of different nations.
- many companies that have innovative products and/or services are located in Massachusetts or New England.

Grades 5 - 8

Solar House Design Project

Your challenge is to design a model house that can be heated efficiently by the sun. You'll be building a model of a solar house for this project using a 150-watt lamp as a model sun.

Design Constraints

The following materials may be used in constructing your model solar house:

- *Up to 3000 square centimeters of corrugated cardboard. The corrugated cardboard must be approximately 4 mm thick.*
- *Plastic wrap (normally used in the kitchen) or acetate (normally used for transparencies for the overhead projector). This material may be used only for windows.*
- *Cellophane tape or glue, only for joining other materials.*
- *1 container to be placed inside the model. Maximum volume: 1/4 liter. It can be filled with any thing you want (water, sand, etc.).*
- *One 150-watt flood light. You may use a piece of cardboard to shield the light from your eyes. You may not add external reflectors.*
- *Paint: You may paint the model and any of its components any color you want.*

Note: The face of the light bulb must be 25 cm away from the model.

Criteria for Evaluation

As you compare your solar house models, more efficient models will be those which:

- *Achieve a greater temperature difference between the inside of the model and the room temperature in your classroom.*
- *Have a slower cooling rate.*
- *Have a larger internal volume.*

Thinking about your solar house design

In designing your model you will need to take into account competing factors. For example, a larger internal volume might reduce the maximum possible temperature in your model. Larger windows might allow more heat in, but they also might permit more heat to escape. Think about it. Perhaps you will decide to set priorities, that is, to consider some parameters more important than others. In any case, do not discard any of the three criteria for evaluation (listed above). Rather, think how you will take them into account.

It is important to keep each of the parameters within a reasonable range. An internal volume that is too high might make it impossible to achieve measurable differences in temperature. Design is a process of exploration. Try using the given materials in imaginative ways. For example, can any of the allowed materials be used as insulation?

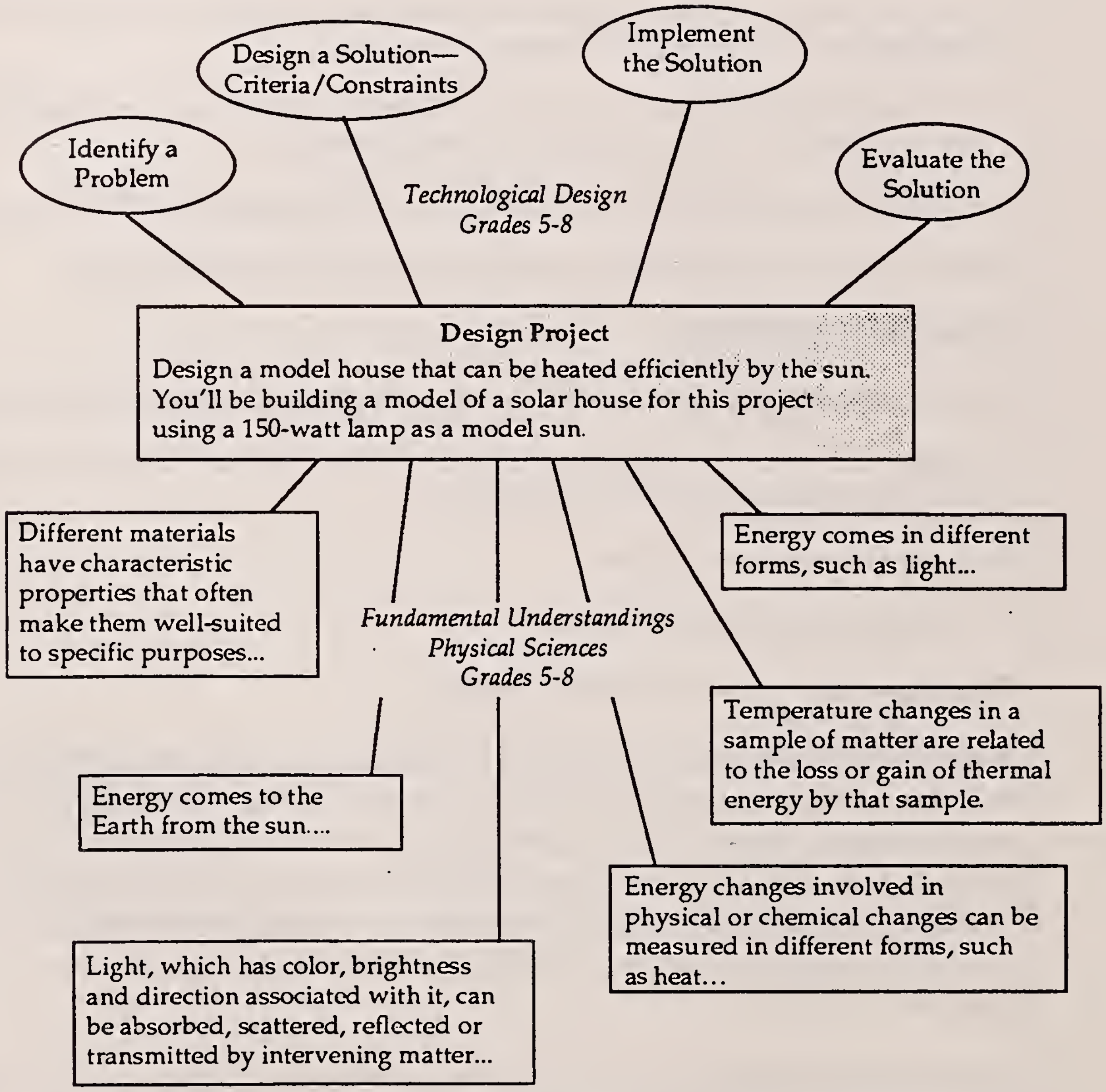
Example of a Technological Design Project

This is an example of a technological design project that could be posed to middle school students as part of their study of alternative sources of energy.

Science and technology are closely intertwined. As pointed out by The Association for Supervision and Curriculum Development, we *"should help students discover these fields of study as the integrated whole they really are." (Loucks-Horsley, 1990, p. 32)* This solar house design project provides the opportunity for students to begin developing fundamental understandings of physical science, while at the same time honing their skills in technological design.

The diagram illustrates the technology and science skills as well as the physical sciences content standards that are addressed in the solar house design project.

Technology and science skills and physical science content standards addressed in the Solar House Design Project

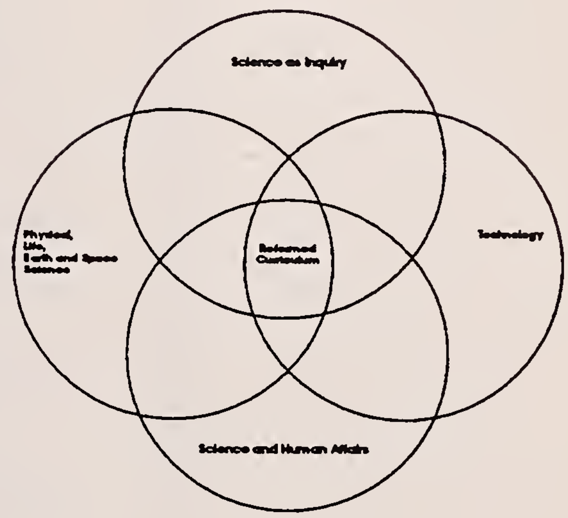


9th & 10th Grade Grouping Content Standards

Inquiry

Students should be able to:

- make observations
- ask questions and use concepts to guide scientific investigations
- design and conduct a full scientific investigation
- use technologies in conducting investigations
- construct and revise scientific explanations and models using logic and evidence
- recognize relationships among patterns
- recognize and analyze alternative explanations and models
- communicate and defend a scientific argument



Human Affairs

Students should be able to:

- understand how science and society have influenced each other in the past and how science has been an integral part of the history of human society
- understand the influence science and technology have on today's society
- recognize that the key problems that scientists address change in response to changing societal pressure and needs
- recognize that while technology can be used to attempt to solve societal problems, technology can also have a negative impact on society and the natural world
- understand how scientific revolutions have changed society
- develop skills in applying scientific knowledge to making decisions about problems at the community, state, national and international levels, and recognize that using these skills responsibly is an essential part of being a citizen in today's world

Vignette to be placed.

Physical Sciences

Physical Science

The physical sciences involve the study of energy and matter. In this framework, the physical sciences include both physics and chemistry. When studying the physical sciences students explore the composition, structure, properties, and reactions of matter as well as the relationships that exist between matter and energy. The developmental progression of students' studies in the physical sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades 9-10 -- By the end of this period students will understand the evidence that underlies a more complex model of matter, including the electronic and nuclear structure of the atom. These fundamental ideas allow students to investigate, explain, and to a limited extent, predict the structure and reactions of simple compounds, making it possible to make informed decisions about a variety of household and industrial processes.

The examination and description of motion becomes more quantitative at this level. The vector quality can be added to the description of motion. Through graphical representations and the gradual introduction of calculus, well-defined laws and principles of motion are introduced. The ideas of conservation of energy and energy transformations should be introduced early in the grade 9-10 stage, and extended toward the end of this stage to include the principle of conservation of energy and the notion of entropy.

Grades 9 &10

Physical Science

Physical Sciences Content Standards
Grades 9-10

Structure of Matter

- The observed properties of elements, each of which is made from a single type of atom, result from the number and arrangement of electrons in their atoms.
- Compounds form when atoms of two or more elements bond. Chemical bonds form when atoms share or transfer electrons.
- Matter is made up of elements, compounds, and numerous mixtures of these two kinds of substances.
- Elements and compounds can be grouped into classes, based on similarities in their structures, and resulting properties.
- Radiation emitted during nuclear changes can affect living materials, (i.e., it can damage cells).
- Energy released in certain nuclear reactions can be harnessed in beneficial ways within a nuclear power plant or released suddenly and destructively in atomic or hydrogen bombs.

Interaction of Substances (Chemical/Physical Changes)

- Solubility of substances may vary with temperature and the nature of the solute and the solvent.
- Balanced electrical forces between charges of the protons and electrons are responsible for the stability of substances. Chemical interactions and physical changes occur when these forces are altered.
- Chemical changes can be explained in terms of rearrangements of atoms, which is made possible by the breaking and forming of chemical bonds.
- Chemical reactions can be represented by symbolic or word equations that specify all reactants and products involved.
- The periodic table is useful in predicting the chemical and physical properties of known elements and those yet to be discovered.

Forces and Motion

- All forces have both magnitude and direction. Forces acting in the same direction reinforce each other. Forces acting in different directions may detract or cancel each other.
- If an object exerts a force on a second object, then the second object exerts an equal and opposite force on the first object.
- All changes in motion or momentum are caused by forces. Magnetic forces act on moving, charged particles. Electrical forces act between charged particles. Friction depends on contact between masses.

Grades 9&10

Conservation and Transmission of Energy

- The total amount of mass and energy remains constant in any chemical or physical change in a closed system.
- Waves, such as electromagnetic waves or sound waves, have wave length, amplitude, frequency, and characteristic speed. Waves can be used to transmit signals or energy without the transport of matter.
- Electromagnetic waves, unlike sound waves, can be transmitted through a vacuum.
- The same concepts of energy, matter and their interactions, apply both to biological and physical systems on Earth and in the observable Universe.

Vignette to be placed.

Life Science

Life Sciences

Life sciences involve the study of living organisms, of life processes and of the different levels of organization from the cell to the biosphere. They include the study of interactions between organisms and between organisms and their environments. Life sciences also address the unity and continuity of life as well as change and diversity. The study of life sciences enhances understanding of human life: how the human body works, develops and changes, and how humans interact with the environment. The developmental progression of students' studies in the life sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

Grades 9-10 -- Students come to grades 9-10 with the fundamental understanding of organisms, ecosystems, and the interdependence of living things necessary to address four of the big ideas of the life sciences:

- the chemical nature of life processes;
- the molecular basis of heredity;
- evolution; and
- the flow of matter and energy in biological systems.

These big ideas are fundamental to the life sciences, but they are also fundamental to the understanding of many of the social and environmental issues with which scientists of our times are concerned.

Molecular biology will continue into the twenty-first century as a major area of science. Already, applications of research in this area have provided people with powerful tools for modifying the living environment to conform to their own ends. Students need an understanding of the chemical basis of life not only for its own sake, but because of the need to adopt informed positions on some of the practical and ethical implications of humankind's capacity to tinker with the fundamental nature of life.

Likewise in the arena of environmental issues, students' understanding of the chemical basis of life, the molecular basis of heredity, and the nature of the flow of matter and energy in biological systems all contribute to the scientific understanding and knowledge that students will need to help make informed decisions about their environment.

Grades 9&10

Life Science

Life Sciences Content Standards
Grades 9-10

Characteristics of Organisms

- Many molecular aspects of life processes of multicellular organisms occur in cells.
- Cells are highly organized collections of chemical substances. The fundamental chemical substances of life are long chains of carbon atoms with differing functional groups. Important among these are: carbohydrates, lipids, proteins and nucleic acids.
- Biological systems — cells, multicellular organisms and ecosystems — obey the same conservation laws as physical systems.

Evolution of Life

- The theory of biological evolution is that the earth's present-day species developed from earlier, distinctly different species.
- Natural selection leads to organisms that are well suited for survival and/or reproduction in particular environments. Chance alone can result in the persistence of some heritable characteristics having no survival or reproductive advantage or disadvantage for the organism. When an environment changes, the survival value of some inherited characteristics that may change.
- Evolution builds on what already exists, so the more variety there is, the more there can be in the future. But evolution does not necessitate long-term progress in some set direction.

Principles of Heredity

- Cells are the repositories of biological information.
- Chromosomes are the components of cells which convey hereditary information from one cell to its daughter cells and from a parent to its offspring.
- Chromosomes are composed of subunits called genes; each gene encodes the information directing the synthesis of a cell product, usually a protein, and can often be identified with a trait observed in the organism. Each chromosome contains a molecule of DNA, a long polymer that encodes information using four different subunits. The structure of DNA, a double helix, insures that the cell can replicate the coded information.
- In reproductive processes involving two parents (sexual reproduction), two specialized reproductive cells (gametes), one from each parent, fuse. One set of chromosomes from each parent is present in the resulting cell (zygote) which directs the formation of a new organism that has attributes of both parents.

Grades 9 &10

Life Science

Matter and Energy in Ecosystems

- The conservation of energy law is a powerful tool for the analysis of energy flow in ecosystems.
- Energy is supplied to ecosystems by sunlight and dissipates as heat.
- Plants convert light energy into chemical energy. High energy chemical substances produced by plants, carbohydrates and fats are the primary source of energy for all animal life.

Vignette to be placed.

Grades 9&10

Earth and Space Sciences

Earth and Space Science

In earth and space sciences, students investigate the origin, structure, and physical phenomena of the earth and study the earth as a part of the universe. Earth and space sciences include areas such as geology, meteorology, oceanography, and astronomy. The study of earth and space sciences deepens our understanding of processes such as plate tectonics, properties of geological materials, changes in the earth's topography over time, and the place of the earth in the universe. The discussion below has been adapted from the NRC's *National Science Education Standards: Fundamental Understandings for Earth and Space Sciences* (August 1993).

Grades 9-10 -- In grades 9-10, students continue their study of the earth in space by expanding that domain to include the universe. Discussion of data from geological and astronomical sources illustrates how scientists use direct and indirect evidence to test the feasibility of alternative theories of the origins of stars and planets (including our own solar system).

Students develop ideas related to the nature of processes and cycles within the earth's system, the role of force and energy (for example, in tectonic plate movement), and the dynamic and interactive nature of the earth as a system as it changes over time.

Grades 9&10

Earth and Space Science

Earth and Space Sciences Content Standards
Grades 9-10

Matter and Energy in the Earth System

- The two fundamental forces acting in the Earth System are gravity and electromagnetism. Gravitational force acts between masses and is responsible for changes in the motion of objects on Earth and throughout the universe. Electromagnetic force holds matter together, and shields Earth from the solar wind.

The Evolution of the Earth System

- The universe is estimated to be over ten billion years old.
- The Solar System is thought to have formed from a cloud of gas and dust that condensed under the influences of gravity and rotation. Most of the mass of the cloud condensed to form the Sun at the cloud's center. The differences among the planets, moons, and other objects in terms of chemical composition and physical state were determined by the distances from the center of the cloud at which they condensed.
- Life is adapted to conditions on the earth, including the force of gravity that enables the planet to retain an adequate atmosphere, and an intensity of radiation from the Sun that allows water to cycle between liquid and vapor.
- The Milky Way is but one galaxy in a vast, ancient, and expanding universe, which contains a tremendous number of galactic clusters. Most of the Universe appears to be empty space.

Geochemical Processes and Cycles in the Earth System

- Rocks undergo changes from both physical weathering (e.g., abrasion, frost wedging) and chemical weathering (e.g., oxidation, solution, hydrolysis) to produce sediment and soils. Climate controls which of these processes dominates.
- The "solid" Earth has a layered structure, with each layer having characteristic composition and physical properties. A solid inner core is surrounded by a liquid outer core, which in turn is surrounded by a large zone of dense mantle material. The crust is relatively thin compared to the other layers of Earth's interior. The layers are interconnected by the transfer of heat and material by conduction and convection. The latter results in the movement of the crustal plates.
- The solid crust of the Earth-including both the continents and the ocean basins-consists of separate plates that ride on a denser, hot, gradually deformable layer of the earth. The crust sections move very slowly, pressing against one another in some places, pulling apart in other places. Ocean-floor plates may slide under continental plates, sinking deep into the Earth. The surface layers of these plates may fold, forming mountain ranges.

Grades 9&10

Vignette to be placed.

Technology

Technology

Investigating the salinity of the water in Vineyard Sound is a pursuit of science. Creating a way to make this salt water drinkable is a pursuit of technology. Simply stated, science is an attempt to understand the natural world; technology is an attempt to create solutions to human problems. Yet science and technology are not necessarily mutually exclusive fields of endeavor. In fact, they are intimately bound together. Scientific understanding provides us with a basis for recognizing, and proposing solutions for, problems of human adaptation. Technology enables us to further our knowledge about the world. The interrelationships between science and technology — and their connections to educational goals — are depicted in the diagram.

People are surrounded by products of the technological design process. As pointed out in the AAAS *Benchmarks*,

> Young children are veteran technology users by the time they enter school. They ride in automobiles, use household utilities, operate wagons and bikes, use garden tools, help with the cooking, operate the television set, and so on. Children are also natural explorers and inventors, and they like to make things. (p. 44)

It is important to capitalize upon these experiences and natural inclinations. Learners should have opportunities to identify problems of human adaptation to the environment and then, to apply problem-solving strategies, design investigations, and build models and physical tools in solving these problems. The content area "Technological Design" emphasizes the importance of creating solutions through the process of design. The design process allows students to identify or discover a problem, apply scientific knowledge and skills in conceptualizing a solution, weigh options, make decisions, construct and test tools, reflect on results, and reformulate their ideas.

Some students are excellent at doing things with their hands but have difficulty verbally representing what they are doing; others have strong verbal skills. The school environment tends to favor students of the latter type. Because design problems involve the construction of prototypes or structures that embody formal ideas, they can stimulate students to transfer knowledge between the two realms.

Grades 9&10

Technology Content Standards
Grades 9 -10

The nature and impact of Technology

- rapidly changing technology affects global competition and jobs; a technology device, service, or system is used for a particular purpose.
- technological impacts can be multidimensional (ie. economic, social, environmental, political).

Technology yesterday, today and tomorrow

- technological development has been influenced by the culture of the society and by the resources available to that society.
- there are new and emerging technologies in areas of communication, construction, manufacturing, transportation, power, bio-related technology.

The tools and machines of Technology

- a wide range of contemporary tools, machines, measuring instruments, computer-based tools, and data-capturing sensors process and measure materials, energy, physical phenomena, and electronic signals.
- a range of complex tools, machines, and equipment are used to solve problems in a variety of technological areas.

Resources of Technology

- there is a wide range of resources including synthetic, composite, biological energy sources and forms of information.
- there are and will be issues on resource management systems such as a waste management system or a nuclear power system that take into account safety, costs, environmental and political concerns.

Technological areas of communication, construction, manufacturing, transportation. power, and bio-realted technologies

- combinations of graphic and electronic communication processes are used in encoding, transmitting and decoding messages.
- the manufacturing processes of forming, separating, conditioning and combining, implement complex products and structures. Manufacturing processes have changed with improved tools and techniques.

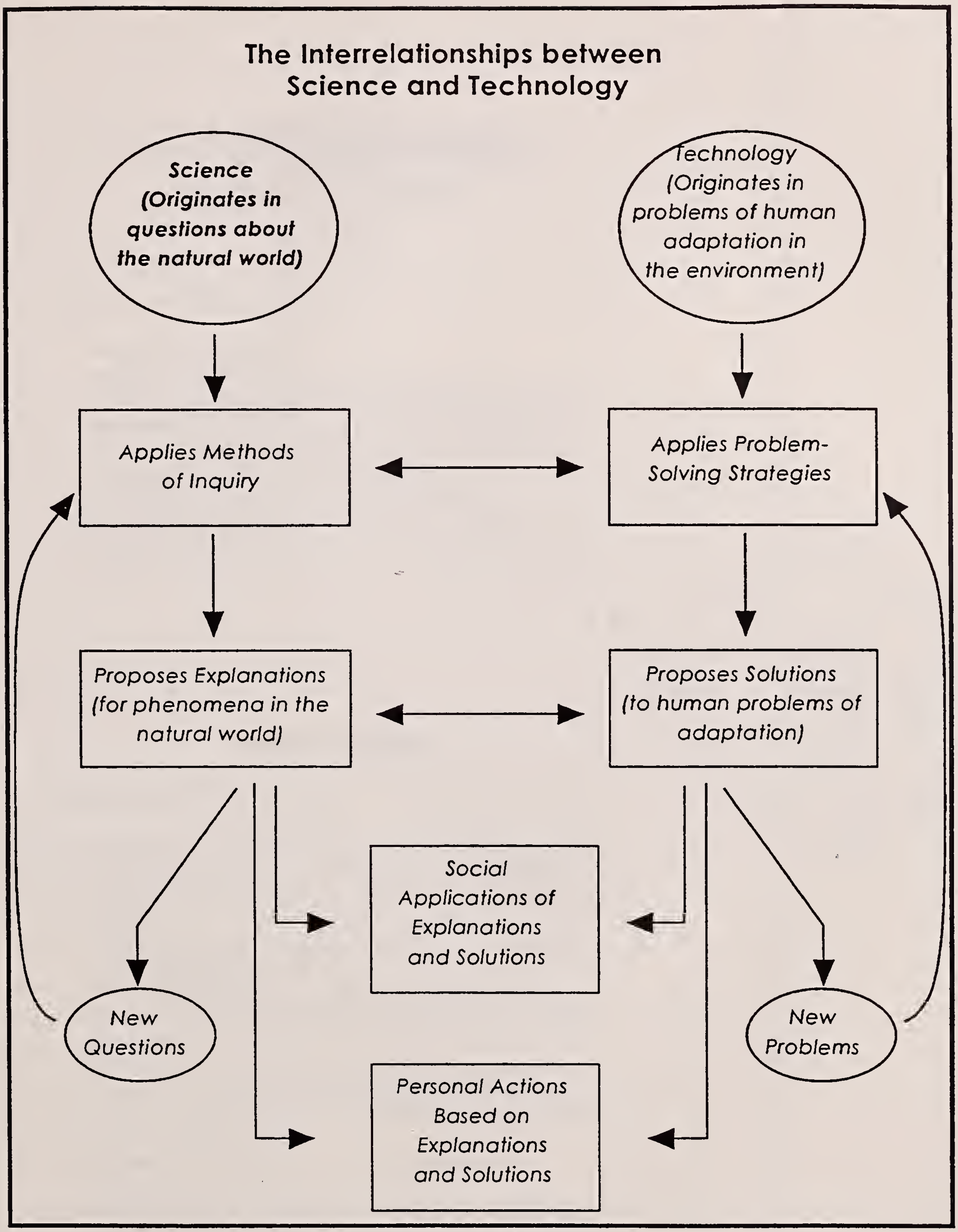
Technology

- existing transportation technologies convey people and products globally.
- a combination of power production categories provide power to address a particular need.
- technological processes could adversely affect the environment, such as a waste management process.

Technology in a Global Society

- technologies do not remain a monopoly for long due to the ease of technology transfer.
- technology has played a key role in the operation of successful U.S. businesses.
- technological inventions and innovations stimulate economic competitiveness, and are translated into products and services with marketplace demands.

Grades 9&10



Grades 9 & 10

Students should be able to:

- Identify a problem
- Design a solution -- *Cost, Risk, Benefit, Analysis*
- Implement a solution
- Evaluate the solution

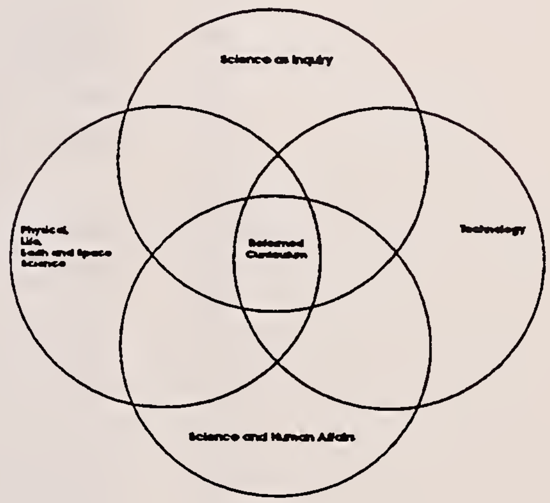
Vignette to be placed.

11th & 12th Grade Grouping Content Standards

Inquiry

Students should be able to:

- make observations
- ask questions for scientific investigations
- design and conduct a full scientific investigation
- use technologies in conducting investigations
- construct and revise scientific explanations and models using logic and evidence.
- recognize relationships among patterns
- recognize and analyze alternative explanations and models.
- communicate and defend a scientific argument.



Human Affairs

Students should be able to:

- understand how science and society have influenced each other in the past and how science has been an integral part of the history of human society
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Physical Science

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Grades 11&12

Physical Sciences Content Standards
Grades 11 -12

Structure of Matter

- A useful model of the atom consists of a positively-charge nucleus (composed of protons and neutrons) surrounded by one or more negatively-charged electrons, held together by electrical forces described in Coulomb's Law.
- Atoms of the same element have the same number of protons and electrons but may differ in the number of neutrons (isotopes).
- The properties of a compound can often be predicted from the structure of its smallest units (either molecules or ionic crystals).
- Pure substances can be represented by formulas or three dimensional models showing the number, types and/or relative positive atoms that make up the substance.
- Forces among particles in a nucleus are extremely strong and act at very small distances; large quantities of energy are associated with nuclear changes.
- Some nuclei are radioactive. The atoms undergo radioactive decay at their own characteristic rate.

Interaction of Substances (Chemical/Physical Changes)

- All observed changes involve either a net decrease in potential energy or a net increase in disorder (entropy), or both.
- Some changes do not proceed to completion, but reach a state of equilibrium with the rate of change in one direction being equal to the rate of change back in the other direction.
- Saturated solutions reach a state of equilibrium.
- Chemical reactions can be classified into general types based on the nature of the changes involved such as acid/base, oxidation-reduction, precipitation, polymerization.
- The rate of reaction can be increased by adding a suitable catalyst. The rate is also affected by changes in temperature or concentration of the reactants.
- The amount of energy involved in changes of state of molecular liquids and solids is determined by the relatively weak attractive forces between the molecules.

Forces and Motion

- The position and motion of an object are judged relative to a particular frame of reference. An object at rest tends to stay at rest unless acted upon by some outside force. An object in uniform motion remains in this state of motion with constant momentum.

Physical Science

- Motion can take place in two or three dimensions. An object's motion can be described in terms of velocity or acceleration, and can be represented in various ways, including distance-time, and speed-time graphs, as well as by mathematical equations and vectors.
- Constant motion in a circle requires a force to maintain it.
- Acceleration is the rate of change of velocity, where the change may be in magnitude or direction. Force and acceleration are related by the relationship F-ma (Force = mass X acceleration).

Conservation and Transmission of Energy

- The total quantity of energy in a closed system remains constant in any chemical or physical change, although its usefulness to prompt further change is reduced through each process as randomness increases.
- Interactions of matter with electromagnetic radiation, electricity or heat can produce useful evidence regarding the structure and composition of matter.
- The loss or gain of thermal energy by a sample of matter is related to a temperature change, which depends on the sample's mass, the nature of its material, and the environment in which it is placed.
- Characteristic and predictable quantities of energy are associated with each chemical and physical change.

Vignette to be placed.

Grades 11&12

Life Sciences

Life Science

Life sciences involve the study of living organisms, of life processes and of the different levels of organization from the cell to the biosphere. They include the study of interactions between organisms and between organisms and their environments. Life sciences also address the unity and continuity of life as well as change and diversity. The study of life sciences enhances understanding of human life: how the human body works, develops and changes, and how humans interact with the environment. The developmental progression of students' studies in the life sciences, discussed below, has been adapted from the NRC's *National Science Education Standards: An Enhanced Sampler* (February 1993).

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- the chemical nature of life processes;
- the molecular basis of heredity;
- evolution; and
- the flow of matter and energy in biological systems.

These big ideas are fundamental to the life sciences, but they are also fundamental to the understanding of many of the social and environmental issues with which scientists of our times are concerned.

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Likewise in the arena of environmental issues, students' understanding of the chemical basis of life, the molecular basis of heredity, and the nature of the flow of matter and energy in biological systems all contribute to the scientific understanding and knowledge that students will need to help make informed decisions about their environment.

Grades 11&12

Life Science

Life Sciences Content Standards
Grades 11-12

Characteristics of Organisms

- Every cell is covered by a membrane that controls what can enter and leave the cell. In all but quite primitive cells, a complex network of proteins provides organization and shape, for animal cells, movement.
- Cell membranes often are the sites of chemical syntheses and energy conversions essential to life. Each metabolic event consists of many chemical reactions, each catalyzed by a specific enzyme. Information about the nature of substances synthesized is carried to the site of the synthesis by a form of RNA.
- In complex, multicellular organisms, cells have specialized functions, communicate with each other and are mutually dependent.

Evolution of Life

- Natural selection provides the following mechanism for evolution: Some variation in heritable characteristics exists within every species, some of these characteristics give individuals an advantage over others in surviving and reproducing, and the advantaged offspring, in turn, are more likely than others to survive and reproduce. The proportion of individuals that have advantageous characteristics will increase.
- Molecular evidence substantiates the anatomical evidence for evolution and provides additional detail about the sequence in which various lines of descent branched off from one another.
- Life on earth is thought to have begun as one-celled organisms about 3.6 billion years ago (Keeton and Gould, 1993). During the first 2 billion years, only singled-cell microorganisms existed, but once cells with nuclei developed about a billion years ago, increasingly complex multicellular organisms evolved.

Principles of Heredity

- Many (50,000-100,000) bits of information, or genes, are encoded in human DNA. The expression of a given trait will depend, to some degree, on the genetic background made up of all other traits.
- In some cases, it is possible for a new organism to grow from a single cell or a cluster of cells from the parent organism (asexual reproduction). In the case of asexual

Grades 11&12

reproduction, the offspring is exactly like the parent.

- The genetic information in the DNA molecules provides instructions for assembling protein molecules. The code is virtually the same for all life forms.
- DNA is a chemical substance that can be separated from cells and altered mechanically and chemically in test tubes. When altered DNA (from the same species) or DNA from another species is introduced into a cell a new trait may be introduced into the cell's genetic material.
- Fragments of DNA can be analyzed to identify the individual from which the sample of DNA came, diagnose human genetic abnormalities, and to study populations.
- Mutations in DNA occur naturally at low rates. When a cell is exposed to radiation or certain chemical substances the probability of mutations increases. If this change occurs in the germ cells, it is passed on to the offspring. If it occurs in other (somatic) cells it is not passed on to the offspring.

Matter and Energy in Ecosystems

- Energy flows through an ecosystem from prey to predator in the form of high energy chemical substances.
- Energy conversions that take place when animals metabolize carbohydrates and fats from plant or other animal sources are inefficient.
- Matter is recycled in ecological systems (water cycle, carbon cycle, nitrogen cycle).
- Herbivores and carnivores ingest only a small proportion of the energy fixed by plants. Most of the energy fixed in an ecosystem flows through its decomposers and detritivores.

Vignette to be placed.

Earth and Space Science

Earth and Space Sciences

In earth and space sciences, students investigate the origin, structure, and physical phenomena of the earth and study the earth as a part of the universe. Earth and space sciences include areas such as geology, meteorology, oceanography, and astronomy. The study of earth and space sciences deepens our understanding of processes such as plate tectonics, properties of geological materials, changes in the earth's topography over time, and the place of the earth in the universe. The discussion below has been adapted from the NRC's *National Science Education Standards: Fundamental Understandings for Earth and Space Sciences* (August 1993).

Grades 11-12-- In grades 11-12, students continue their study of the earth in space by expanding that domain to include the universe. Discussion of data from geological and astronomical sources illustrates how scientists use direct and indirect evidence to test the feasibility of alternative theories of the origins of stars and planets (including our own solar system).

Students develop ideas related to the nature of processes and cycles within the earth's system, the role of force and energy (for example, in tectonic plate movement), and the dynamic and interactive nature of the earth as a system as it changes over time.

Grades 11&12

Earth and Space Sciences Content Standards
Grades 11-12

Earth and Space Science

Matter and Energy in the Earth System

- Energy is transferred through the Earth System by three mechanisms: radiation, conduction, and convection. Earth receives energy from the Sun in the form of ultraviolet, visible light, near and far infrared radiation, and radio waves, and reradiates this energy to space as far infrared radiation. Conduction is responsible for the transfer of earthquake waves through Earth's interior, and of energy and materials between Earth's surface and atmosphere. Convection moves energy through the Earth System in the form of winds, ocean currents, and movements inside Earth's mantle and crust.

The Evolution of the Earth System

- Increasingly sophisticated technology is used to learn about the universe such as visual, radio, x-ray telescopes, computers, space probes, and accelerators give subatomic particles energies that simulate conditions in the stars and in the early history of the universe before stars formed.
- Stars vary in size, composition, mass, surface temperature and luminosity. The light received from a star contains information from which these quantities can be estimated. This has made it possible to determine the source of energy within stars and to construct models for how stars evolve. Current understanding of the evolution of stars suggest that for most of their lifetimes, most stars gradually change in size, surface temperature, and luminosity as they consume their nuclear fuel.

Earth in the Universe

- Earth's global climate is determined by the planet's distance from the Sun, the tilt of its spin axis with respect to the plane of Earth's orbit, the length of the planet's day compared to the length of its year, the composition of its atmosphere and oceans, and the properties of its surface. The seasons are due to the tilt of Earth's spin axis with respect to the plane of its orbit around the Sun. This causes unequal heating of Earth's surface by the Sun's energy, varying with latitude and with time of year.
- Mathematical models and computer simulations are used in studying evidence from many sources in order to form a scientific account of the universe.

Geometrical Processes and Cycles in the Earth System

- Earthquakes and volcanoes often occur along the boundaries between colliding plates, and molten rock from below creates pressure that is released helping to build up mountains.

Grades 11&12

Earth and Space Science

- The global climate varies on several time scales in part due to the changing energy received from the Sun. The energy output of the Sun has varied slightly over periods of hundreds to thousands of years. Additionally, on time scales of tens of thousands to hundreds of thousands of years, the intensity and distribution of the energy received by the planet has varied as a consequence of Earth's continually changing orbit.
- On a planetary scale, both matter in the form of water, and energy in the form of latent heat, circulate through the Earth System in the water cycle. Over a period of a few days, water vapor rises from Earth's surface into the atmosphere. Winds carry the vapor poleward where it condenses to form clouds and precipitation, releasing the energy initially used to evaporate the water. Over several weeks water flows down a river to the sea. Over many years water flows through underground aquifers.
- Carbon, sulfur, nitrogen, and other elements are continuously cycled through the Earth System. These cycles involve both the global distribution of these elements and their compounds among various reservoirs, and the rate at which these substances transfer between reservoirs through chemical reactions. The biosphere is an important component of many of these cycles.

Vignette to be placed.

Grades 11&12

Technology

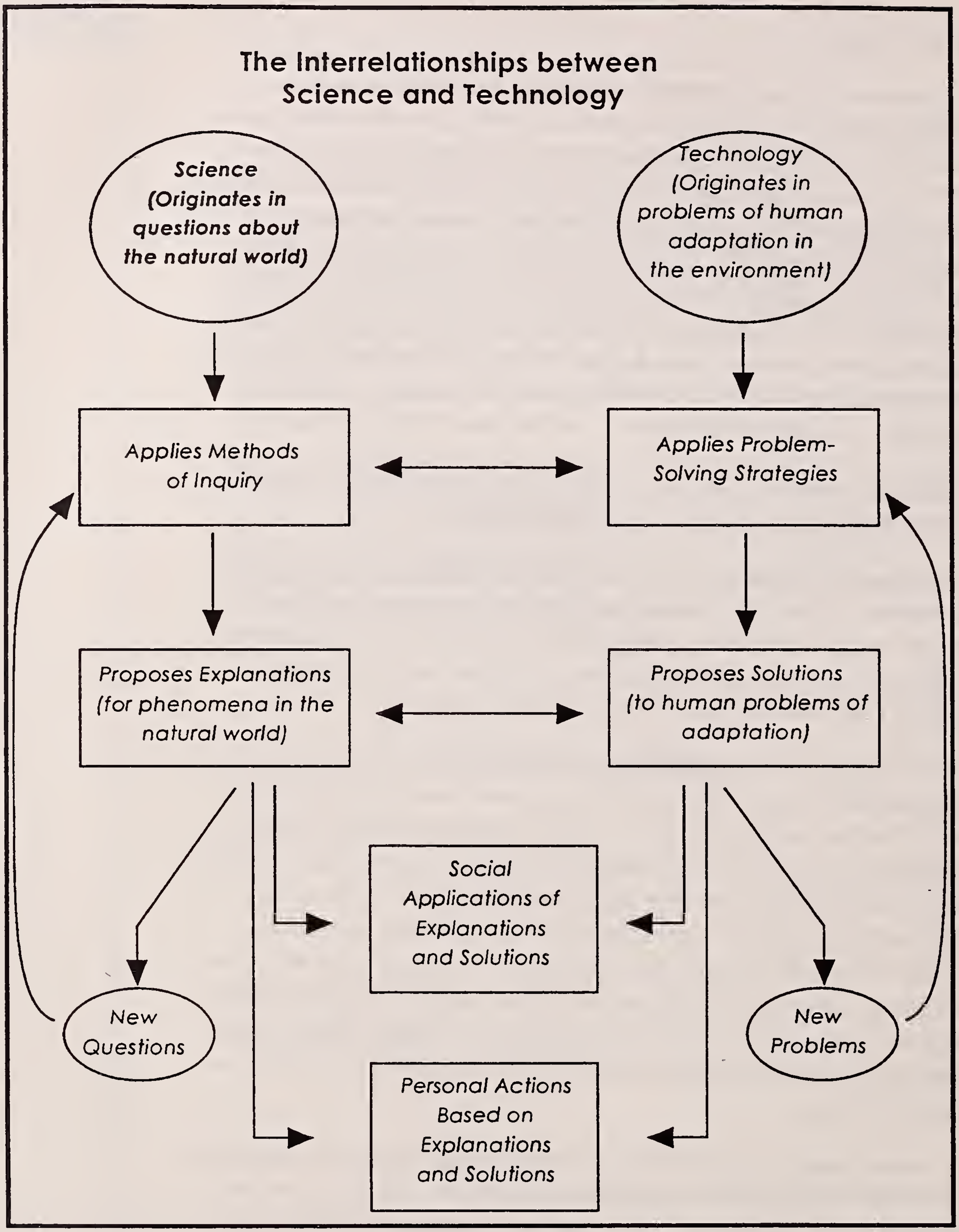
Investigating the salinity of the water in Vineyard Sound is a pursuit of science. Creating a way to make this salt water drinkable is a pursuit of technology. Simply stated, science is an attempt to understand the natural world; technology is an attempt to create solutions to human problems. Yet science and technology are not necessarily mutually exclusive fields of endeavor. In fact, they are intimately bound together. Scientific understanding provides us with a basis for recognizing, and proposing solutions for, problems of human adaptation. Technology enables us to further our knowledge about the world. The interrelationships between science and technology — and their connections to educational goals — are depicted in the diagram.

People are surrounded by products of the technological design process. As pointed out in the AAAS *Benchmarks*,

Young children are veteran technology users by the time they enter school. They ride in automobiles, use household utilities, operate wagons and bikes, use garden tools, help with the cooking, operate the television set, and so on. Children are also natural explorers and inventors, and they like to make things.

It is important to capitalize upon these experiences and natural inclinations. Learners should have opportunities to identify problems of human adaptation to the environment and then, to apply problem-solving strategies, design investigations, and build models and physical tools in solving these problems. The content area "Technological Design" emphasizes the importance of creating solutions through the process of design. The design process allows students to identify or discover a problem, apply scientific knowledge and skills in conceptualizing a solution, weigh options, make decisions, construct and test tools, reflect on results, and reformulate their ideas.

Some students are excellent at doing things with their hands but have difficulty verbally representing what they are doing; others have strong verbal skills. The school environment tends to favor students of the latter type. Because design problems involve the construction of prototypes or structures that embody formal ideas, they can stimulate students to transfer knowledge between the two realms.



Grades 11 & 12

Students should be able to:

- Identify a problem
- Design a solution -- *Cost, Risk, Benefit, Analysis*
- Implement a solution
- Evaluate the solution

Technology Content Standards
Grades 11 & 12

The nature and impact of Technology

- technology has both positive and negative impacts on society.
- technology influences business and government policies and actions.
- technology will change in the future and will impact individuals, family and society.

Technology yesterday, today and tomorrow

- there is a process through which an invention is patented, products or processes are patented by males and females of various cultures and races.
- some technologies will become obsolete in the next two to five years and emerging technologies in the areas of communication, construction, manufacturing, transportation, power, bio-related technology will replace them.

The tools and machines of Technology

- a wide range of tools an instruments are used to analyze, adjust, and maintain mechanical, electronic, hydraulic, pneumatic, and electrical systems.
- contemporary problems in the areas of communication, construction, manufacturing, transportation, power, bio-realted technology are resolved through the research and use of technology.

Resources of Technology

- there are benefits and costs associated with the use of resources in technological ventures.
- technology assessment analyzes the properties of resources, their availability, the ease of processing and disposal, and economic considerations.

Technological areas of communication, construction, manufacturing, transportation. power, and bio-realted technologies

- messages may be designed and communicated containing integrated written, audio, and video portions. The purposes of communication may be for information dissemination, persuasion, entertainment, and/or education.

Technology

- automated manufacturing processes have transformed industries.
- an automated continuous manufacturing enterprise involves design, organization finance and a combination of material conversion processes to produce products.
- contemporary and nontraditional construction practices (e.g., fabric structures, geodesic domes, foam structures) are used to create a structure.
- intermodal transportation areas transport people and products efficiently with minimal risk to the cargo and to the environment.
- industrialization brings an increased demand for energy usage and also leads to more rapid depletion of the Earth's energy resources.
- efficient use of technology can slow the rapid depletion of the Earth's energy resources.
- technology forecasting determines the effects of various bio-related developments such as increase plant growth, water purification systems.

Technology in a Global Society

- new management techniques change and affect the workplace.
- globalization causes problems with issues such as standardization, copyright, patent infringements, value added cost, etc.
- technology plays a major role in making a country a world power.

Vignette to be placed.

Grades 11&12

Conclusion

For many districts, realizing the vision of science education presented in this framework will take time, resources, collaborative planning, and commitment. The numerous issues and practices involved in systemic science and mathematics reform are addressed in depth in the next chapter of this framework. Presented here are some implementation issues of particular relevance to science education, including the need for appropriate facilities and materials, curriculum coordination, and legal responsibilities.

Facilities and materials. Districts should work toward ensuring that students have the facilities and materials needed for undertaking scientific investigations in elementary, middle, and high schools. The facilities should include sinks, outlets, storage space for equipment and supplies, tables or other large surfaces where students can work, and ample areas where students can keep their projects for continued use over a number of classes. Fire extinguishers and first aid kits should be readily accessible. It is essential that students have appropriate quantities of materials and equipment in order to do hands-on, inquiry-based science.

Curriculum coordination. It is important that a district's science program be viewed as a whole so that the scope and sequence of the program from Pre-K through twelfth grade is coherent. The district science coordinator should be involved in articulating and coordinating district-wide (K-12) science programming. In addition, a science coordinator for the elementary grades could help to ensure that teachers in elementary schools are supported in their efforts to help students learn science.

Legal issues. Administrators and teachers should know the Massachusetts laws which are relevant to science education. These include regulations regarding safety, use and care of animals, and disposal of hazardous waste. Information about these regulations is included in Appendix 1.

Chapter 4: Implementing the Frameworks

The publication of these Curriculum Frameworks represents a profound challenge to educators in Massachusetts: the challenge is to provide an effective mathematics and science education for all students in the Commonwealth. The development, review, and dissemination of the frameworks for Mathematics and for Science and Technology was an essential step. It is, however, only the beginning. Implementing the frameworks throughout the state will require systemic reform. That is: 1) the contributions of all stakeholders will be essential; 2) it will be necessary to plan and monitor the change process in each district; and 3) simultaneous changes will be needed in all relevant aspects of schools and school programs.

This is not a minor task. Nor is it a single event. The implementation of new mathematics and science programs throughout the Commonwealth will be an ongoing process which takes place over several years. It will require understanding and planning for change, and providing effective professional development for teachers and administrators. It will also require continuing curriculum modification, careful selection of instructional materials, development of authentic assessments, and review of school and district policies.

No one group of people will be able to make the changes which are needed in the public schools of the Commonwealth. This effort will require a broad consensus and a joint commitment by all potential contributors. Through earlier efforts to provide quality education for all students, educators have learned that focusing on one segment of the school community at a time often leads to short-term, hence ineffective, efforts. Implementing the Frameworks will require a coordinated effort on the part of school committees, school councils, superintendents, principals, teachers, and parents. In addition, this systemic reform will require partnerships outside the immediate school community—with colleges and universities, museums, cultural institutions, businesses and industries, and other community groups and institutions.

This chapter is organized into three major sections. The first section, Partnerships, focuses on some possible partnerships and roles for partners with school districts. The second section, Planning and the Change Process, addresses some of the critical factors that school systems need to consider for improving their programs. The third section, Systemic Reform within School Systems, discusses the importance of

professional development, offers suggestion for reviewing curriculum and selecting instructional materials, and also discusses how schools might begin to examine issues related to equity.

Partnerships

"It takes a whole village to raise a child." This ancient African proverb evokes an image of traditional societies—societies in which child rearing was a collective task. This task was deemed to be far too important and complex to leave to any single family or social institution. The proverb reminds us that there are societies in which every individual takes responsibility for the care and education of the young. In modern industrial societies, however, much of this sense of collective responsibility has been lost. Very few people live in villages where everyone is known on a first-name basis. Nevertheless, there are some modern industrial societies where a sense of collective responsibility has been maintained. Consider, for example, this experiment which was recently carried out in a Japanese city.

> *A student was asked to leave the school during the school day and walk through the city. The student was very reluctant despite reassurances that there would be no repercussions for this action. Within a short period of time and a short distance from the school, he was stopped several times and asked why he was not in school.*

The chance of replicating such an experiment in the United States seems slender indeed! Rather than feeling that their educational endeavors are supported by the community and endorsed by the broader culture of our society, teachers in the U. S. often feel they are fighting a solitary battle. At the same time, the importance of a sound education in science and mathematics is being stressed more and more by employers, economists, politicians, and even journalists.

The improvement of mathematics and science education is so critical to our common good that it must be a collective enterprise, with a broad base of support.

Partnerships with Parents

From district to district, the nature of the learning partnerships that a school establishes with its community will differ. Yet there will be one group present in all these partnerships—parents. Parents must come to believe that their highest hopes and expectations for their children are shared by the school, that they are, in fact, working

towards the same goals. Collaboration in a partnership with schools will make these goals easier to achieve.

The publication, *For Our Children, Parents and Families in Education* (National PTA, 1992), points out that parents play a number of roles within schools. They are their children's first and continuing teachers. They may also be involved with schools as resources schools and as clients.

Parents as teachers of their own children

Parents' first interest, and first responsibility, is as teachers and advocates for their own children. They make important contributions to their children's learning by maximizing their children's opportunity to learn from daily events in the home, by taking their children on trips to expand their opportunities to learn, and by supporting and monitoring their children's homework.

In order to do these things most effectively, parents need information and support from the school. They will benefit, for example, from a thorough explanation of the school curriculum, suggestions of ways to support their children's learning at home, and information about teachers' expectations and homework policies. Teachers can help parents by:

- encouraging them to visit the classroom to observe and discuss students' interests, assignments, and skills in mathematics and science,
- suggesting activities that reinforce mathematics and science topics discussed in school,
- providing information regarding mathematics- and/or science-related publications and television programs, and
- extending mathematics and science instruction to the home by designing homework that requires family involvement.

Parents working with and for schools

In addition to supporting the education of their own children, parents offer tremendous potential for supporting and enhancing educational programs in schools. For example, they can serve effectively as:

- advocates for their children and themselves within schools,
- advocates for schools in the larger community, and
- partners in school governance and decision-making.

Teachers may reach out to parents by inviting them to support mathematics and science programs. For example, parents could be invited to collect or supply materials for hands-on activities, to share information regarding their own mathematics and/or science-related careers or hobbies, or to work as volunteers with small groups of students in the classroom, the laboratory, or on field trips.

In order for these interactions to work well, however, teachers will need to orient parents to their work with students. They will need to establish guidelines for confidentiality, explain the learning goals of the lessons, outline the classroom procedures and policies, and explain and model effective ways of interacting with students.

Parents as learners benefiting from school programs

The relationship between schools and parents will be most effective if it is multi-faceted and two-directional. As well as benefiting from the contributions parents make to schools, schools can provide important resources to parents.

Immigrant parents, for example, will benefit from English classes and a chance to meet together to discuss their experiences as parents in a new culture—a culture in which their children may be adapting more quickly than they are. Such programs also offer teachers and school administrators opportunities to deepen their understanding of parents and students from other cultures. As one Cambodian parent wrote:

> *Cambodian parents encounter new social, cultural, and economic problems in the United States. In the face of these problems parents and other adults encourage children to study hard in school and get as much education as possible, in order to be better prepared for their future in this country.*

Parents such as these are ready to entrust the education of their children to the teachers and the schools. They do not see it as appropriate to criticize the U. S. system of education—if the educational system were not good there would not be technical progress, a high standard of living, and a modern way of living. In their view, teach-

Vignette: Parental Involvement in Monson

Monson is a small rural community in western Massachusetts in which the total K-12 student population of 1250 is housed in three school buildings. Several years ago one school principal and the PTA president initiated a systematic plan for including the community in a program of school improvement.

They began by forming a family/community involvement team which included the principal, a few parents, a few teachers and the director of the local senior center. After identifying the local education stakeholders—parents, extended families, senior citizens, and business and community leaders—the committee developed activities to support the roles identified in the national PTA publication—For Our Children, Parents and Families in Education. Within three years they developed a wide variety of activities involving people throughout the community. Currently their activities include the following.

Communication. A monthly parent letter is written jointly by the principal and PTA president. It consistently thanks parents for their ongoing support and suggests home-based learning activities. A district newsletter provides information on what is happening in the school system. The newsletter is written, produced, and distributed by PTA volunteers with help from school staffs, a local newspaper owner, and the high school guidance department. It goes to the families of all students, and is broadly distributed throughout the community as well. Each year a handbook/calendar is produced. One section lists school policies, another lists the special events of the school year. Throughout the calendar are short suggestions to parents—activities a child and parents might enjoy, advice on child rearing, and tips about establishing an effective home-school connection. The local papers and cable TV channel also help to publicize and highlight school events.

Parental education and participation in decision making. A grant provided funds for a program to educate parents in the "new methods" teachers were using—cooperative learning, integrated curriculum, and technology in the classroom. Parents also had opportunities to learn to use computers for their own purposes. Parents are also invited to join committees within the school, on topics such as discipline, awards, organizational and study skills, school goals, and curriculum.

Creating a welcoming environment. PTA members welcome new families by volunteering at preschool and kindergarten screening. They serve refreshments, show parents around the building, and have toys available for siblings. They continue their welcome on the first day of school, and help children and parents find classrooms. At back-to-school night each fall the superintendent, principal, and PTA president welcome the parents. Teachers then meet with parents by grade level to discuss the curriculum and teaching methods, followed by meetings in each classroom.

Engaging all stakeholders and appreciating their contributions. Strong connections have been developed with seniors in the community. Grandparents' day is celebrated each year with visits to classrooms and includes senior center residents as well as grandparents of children in the school. Seniors also serve as volunteers in the school, as penpals, and as hosts when classes visit the senior center.

A system-wide business partnership has been developed, which funds grants to teachers for innovative ideas. Members of the partnership also visit classrooms to talk about their work, and sponsor career awareness days. They are planning to develop internships and work study programs.

The contributions of the community are welcomed and appreciated within the school system. One indication is the PTA volunteer room—a place for volunteers to meet, work, and share their ideas. It includes toys for children accompanying a parent, and a bulletin board with information of interest and importance to parents.

ers are trained, have credentials, and, therefore, know about teaching and learning.

When schools are viewed as contributing to the community as well as to the education of young people, parental support and community support for schools is likely to be enhanced. In every school, students and their families are the number-one stakeholders.

Partnerships with Colleges and Universities

As school systems throughout the Commonwealth of Massachusetts begin to design and implement curricula which are based on these Frameworks, the NCTM Standards, and NRC national science standards, both the content and the way mathematics and science are taught will change substantially. Colleges and universities can make important contributions to this effort.

Inservice education partnerships

Colleges and universities can play an important role in the professional development of teachers. While many school districts have called upon individuals who are members of college and university faculties, there have been too few systematic efforts to develop ongoing professional development programs between schools and colleges and universities. Colleges and universities can provide opportunities for teachers to learn more and to deepen their understanding of mathematics and science. In addition they can assist teachers to reflect on and improve their teaching and assessment practices.

Vignette: Courses for College Credit Offered at the School Site

Sometimes an individual teacher initiates a collaboration between a school system and a college. Jim Wood, a teacher at Tyngsboro High, was concerned that some in-service courses in his system were not available for credit. He was unable to change the policy within the school system, so he contacted Michelle Zido, a colleague at Fitchburg State College. Working together, they arranged for courses to be taught in Tyngsboro with credit available from Fitchburg State. In some instances they have obtained Commonwealth Inservice Institute grants to cover the cost of courses. In other courses teachers pay a tuition fee.

This collaboration has resulted in increased enrollment in in-service courses in the Tyngsboro school district, with most teachers choosing to take the courses for credit.

Vignette: A Professional Development School

Because members of the faculty of Reading High School see educational reform as a necessity they chose to become a Professional Development School. The school entered a formal alliance with UMass Lowell and, with the university, began to develop and articulate a shared vision to restructure teacher education, improve teaching, and enhance student learning. Together, they aim to develop a new model of teacher education, promote professional development, and encourage dialogue among teacher educators, novices, and veteran teachers.

Faculty members of the school and university are working together to promote professional development and collegiality within the high school. They polled the school faculty to determine topics which would be of interest and then met to share ideas and develop mutually beneficial projects. They plan, for example, to design interdisciplinary activities for high school students related to environmental problems. The students will examine environmental problems from the perspective of mathematics, science, history, economics, political science, and literature. University students will assist the high school students as they choose environmental problems to study and as they prepare to create their own multimedia presentations.

In this partnership, Reading High School teachers also contribute to the program for education students at the university. During the pre-practicum, for example,

There are many ways of establishing mutually beneficial partnerships between school systems and colleges and universities. Some Massachusetts schools, for example, are forging strong partnerships with colleges by becoming professional development schools; other partnerships support the continuing professional development of teachers. The following two vignettes illustrate how teachers at two Massachusetts schools have developed partnerships with colleges and universities.

Pre-service education partnerships

One of the most significant contributions universities and colleges can make to the improvement of mathematics and science learning in Massachusetts is through the pre-service preparation of teachers of mathematics and science.

If new teachers are going to be prepared to implement the ideas from these Frameworks in their teaching, it is critical that they have a sufficient number of courses in the mathematics and science content they are expected to teach. This is especially important for students preparing to teach in grades K-4 since few are mathematics

or science majors. Appendix 1 of the Mathematics Framework includes a set of standards, "Recommendations for the Mathematical Preparation of Teachers of Mathematics," developed by the Mathematical Association of America (MAA, 1991).

Not only will future teachers need more and different mathematics and science content, but the way they are taught will also need to change. If they are expected to use technology such as graphing calculators or computers in their classrooms, they must have experiences with these tools in their own courses in college. If teachers are expected to have students become problems solvers, then they must learn to become problem solvers themselves in their college classes. If they are expected to encourage students to use cooperative learning and other non-lecture teaching methods in their classes, then they must experience these teaching methods in their own undergraduate mathematics and science classes. In 1991, the NRC's Mathematical Sciences Education Board prepared a report entitled *Moving Beyond the Myths: Revitalizing Undergraduate Mathematics*. This report suggested the following steps for establishing an action plan:

- Engage mathematics faculty in issues of teaching and learning.
- Teach in a way that engages students.
- Achieve parity for women and minorities and the disabled.
- Increase the number of students who succeed in college mathematics.
- Ensure sufficient numbers of school and college teachers.
- Link colleges and universities to school mathematics.

Since for the next few years, most students entering college will not have experienced the kind of curriculum advocated in these Frameworks, it is especially important that their college experience prepare them to implement both the recommended content and the recommended pedagogy.

There is also a need in pre-service education to assure that prospective teachers learn relevant strategies and methodologies for classroom assessment (Stiggins, 1988). These courses must emphasize formative assessment strategies that will enable teachers to utilize a variety of assessment methods appropriately while adapting them to the specific needs of the content area, objectives of curricula and student developmental level. Several researchers have outlined what should be covered in such a course (Stiggins 1988, Airasian 1991, Schafer 1991).

Partnerships with Museums

Museums can provide learning experiences that enhance and extend classroom activities. A museum is an excellent setting for developing students' skills in observing, communicating, and comparing phenomena. Carefully planned experiences in this context also provide opportunities for developing students' curiosity, creating skepticism, and promoting self-esteem. If a museum field trip is to support and extend classroom experiences, it needs to be carefully planned. Voris (1986) suggested that:

- There should be sufficient pre-trip planning by the museum staff and the classroom teacher.

Vignette: A School/museum collaboration

Mrs. Jones, who teaches both science and mathematics to her sixth grade class, is beginning a unit on models. She wants to work with her students on proportion and scale and decides to take a trip to the Museum of Science to introduce these ideas to her class. Although Mrs. Jones is familiar with the museum, she has never used it to study models, so she visits the museum before the field trip. (Being a Massachusetts teacher, she knows she will be admitted without charge.) Mrs. Jones finds the museum full of models—living organisms and mathematical, physical, life-sized, and scaled models—everything from mollusks to machines.

Before the trip, Mrs. Jones introduces the concept of models to the class and asks her students to make a list of familiar models. She asks the students to think of models they have seen in the museum and continues the discussion by posing questions such as: What are the differences between models and real objects? Can models be misleading? When are scale models useful? Is there a change in structure when a model is scaled down or up?

At the museum, students are asked to find examples of: a real object, a larger-than-life model, a life-sized model, a smaller than life model, a physical model, and a mathematical model. Students are then asked to create lists of what the model does and doesn't tell them about the real thing and to respond to a series of questions: Does the model suggest something that might not be true? (i.e. a globe of the earth in which the height of the mountains is in a different scale than the diameter of the planet?) Why is the model portrayed in a particular scale? Can the scale be misleading?

Back in class, the students compare their findings and discuss their observations and ideas. At the end of the unit, each student is given the same amount of clay and asked to create three scale models of an object: life size, one-half life size, and twice life size. Mrs. Jones uses this project to assess the students' knowledge and understanding of models and scale.

Mary Dussault and Susan Sunbury

- The classroom teacher should prepare the students for the visit and carefully coordinate the visit with the science curriculum.
- The museum experience should have a specific focus and provide active student involvement through a structured set of learning activities.
- The trip should be followed up with activities in the classroom which build on the visit and extend the ideas fostered in the museum.

Partnerships with Business and Industry

The business community and associated professional organizations provide a rich resource for extending science and mathematics programs. Class visits to facilities where science and mathematics are applied can serve to motivate students and help them see the connections between their school studies and the larger world of science and mathematics. In addition, people in the community who work in mathematics and/or science-related occupations are frequently interested in public education, and willing to visit classrooms to discuss their work with students. Role models can generate student interest in careers in mathematics and science. It is especially important that such role models include representatives of groups traditionally under-represented in mathematics and science, such as women and people of color.

Many leaders within corporations in the U. S. are aware of the symbiotic relationship between business and industry and education. They know, for example, that corporations spend $30 billion per year to provide remedial training for employees; and that this money, spent to "plow the same field twice" could be better used for direct subsidy to schools. They are often interested in establishing formal relationships with schools, such as the one described below. There are, for example, currently more than 140,000 education partnerships sponsored by corporate America.

As a sense of corporate responsibility continues to develop, there will be opportunities for school systems to collaborate with businesses developing programs that, for example:

- grant corporate personnel social service leave to work in public school systems,
- furlough retired engineers, scientists, administrators, and technicians to adjunct teaching positions in public schools,
- give financial sponsorship to send teachers to educational seminars,
- expand staff exchange programs between education and industry,
- bring additional direct financial resources to education,

Vignette: Partnerships with businesses

Joan Ashley, a third grade teacher in Attleboro, sat down next to Tricia Keane, training coordinator for Jostens' Attleboro facility, at a Chamber of Commerce meeting in the spring of 1991. Dr. Joseph Rappa, Superintendent of Schools, Gerald Keene, Chairman of the Chamber of Commerce, with other school and business leaders, had gathered to discuss how they could work together to enhance the education of Attleboro's students. As Joan and Tricia became engaged in their own conversation, they found they both had a strong, common belief: Children must come to understand, at an early age, the importance of staying in school. They talked about the class rings that Josten created in Attleboro and how they were a symbol of an essential educational accomplishment for a young adult.

From this conversation, an incredible two year partnership developed. During the summer of 1991, Joan and Tricia spent a few days together at the Josten's facility. They began to discover a myriad of connections between Jostens and Joan's third grade class. Team work and quality products are essential in each. Basic values such as honesty and hard work were needed to assure best working conditions. Together, Joan and Tricia sketched out a plan to develop the partnership over the ensuing school year.

Some of the Activities that resulted from this partnership include:

-Artisan-a-Month: Each month, an employee from Jostens would share the special tools and skills with the students. One month, the children saw the largest Super Bowl ring ever, a size 23 worn by the Chicago Bears' William Perry. Another month, a gem setter brought stones for students to study. This began investigations into a wide range of earth science topics.

-team management activities shared between Jostens and the third graders

-a time clock was given to the class with appropriate cards. The children learned military time and punched in and out, recording weekly time on Fridays

-science unit - Work, Force, and Machines, connecting classroom investigations with actual workings of the Josten's plant

-formation of "Jr. Jostens," a small business modeled after Jostens. Profits were used to buy food for a local food pantry. The students learned and practiced the concepts of inventory, supplies, expenses, profit, sales, taxes, and advertising.

Partnerships such as this provide a meaningful context for students to learn the importance of education in their lives.

Nancy Sprague, PALMS Specialist, Attleboro Public Schools

Vignette: Collaboration between schools and employers

The Fenway Middle College High School is a Boston public high school which is located on the campus of Bunker Hill Community College, and serves students in grades 10 through 12. Fenway's curriculum is academic rather than vocational, nevertheless, the program includes career development, community service, and a strong work-based learning component. One of the three work-based learning sites at the Fenway is based on a partnership with the Children's Hospital. The Fenway-Children's Collaborative aims to provide students at-risk of dropping out of school with the motivation to graduate and pursue a higher education.

The program looks for students interested in health careers whose motivation for academic studies might be improved by a work-based approach. Approximately 30 percent of Collaborative students are classified as special education students, and there are no prerequisites or behavioral criteria for admission to this program

During the 11th grade, work experience is a key component of the final semester (or fourth "marking period"). Every school day of this eight-week marking period, students go to school from 8:30 to 12:30 and to the Children's Hospital for internships from 1:30 to 4:30. During the internship, each student will rotate through four departments, spending approximately two weeks in Patient Accounting, Medical Records, Nutrition and Food Service, and Clinical Laboratories.

During the third marking period of Grade 12, students spend all day, every day, at Children's Hospital doing an internship in one department. Each student is paired with a different supervisor. Because the Collaborative's purpose is to help students who need it, it is not consistent with the program's goals to deprive students of their entitlement to hospital work, even if problems with school or work performance exist. Students are offered help, rather than being pushed out of the Collaborative when they are not performing well.

Graduates continue to college or accept jobs in the hospital. In 1989–90 the dropout rate for Fenway Middle College High School was 12 percent, compared to nearly 40 percent in the Boston school system as a whole. In 1990–1991 the dropout rate was 8 percent at Fenway, compared to 33 percent system-wide.

- provide resources and personnel,
- arrange school-business partnerships,
- assist in curriculum development,
- provide field trip programs, and
- provide career counseling to students.

Newspapers, magazines, and radio and television stations in particular can contribute to this state-wide effort by informing the public about the Frameworks and reporting educational success stories.

Clarifying Roles in Systemic Change Partnerships

As schools and school systems reach out to form new partnerships and relationships, it will be helpful to discuss and clarify the roles of the groups of people who will contribute to the effort to provide effective mathematics and science programs for all students. Principals and teachers, for example, will benefit from sharing their views of their own roles and those of the district administrators and staff. Appendix 2 provides a series of checklists which may help decision makers to define and clarify the contributions each group might make to this important effort.

Planning and the change process

The second major aspect of systemic reform is planning and monitoring the change process within each school district. Because schools and school systems are complex and stable systems, it will be important that everyone involved understands the change process.

Implementation of these frameworks will require changes in the curriculum, in our approach to teaching, in the way we assess students as well as the way we assess programs and schools, and it will require changes in our fundamental assumptions about students, how they learn, and what it means for them to know something. The significant changes which are described in these curriculum frameworks can not be fully implemented in two or three years. They involve seeing reform as an ongoing process, so that progress is continuous and natural. It does not represent a break, but a forward movement.

Change will occur differently in different systems. Systems differ in size, in the degree of change which has already occurred, and in available partners. In each case, however, change will require a sustained effort on the part of all segments of the larger school community—school committees, superintendents and administrators, teachers, parents, and partners. In addition, as changes occur in earlier grades, student will enter higher grades with different skills, attitudes, and knowledge—making further adaptations possible and necessary.

Mobilizing support for change

One important task will be to develop and sustain interest in and commitment to this long-term effort. Among the people in each school and school system there will be varying degrees of interest in and

readiness for making significant changes. There will be teachers and administrators who have already begun this change process. There will be other teachers and administrators who have not yet begun the process, and who will not begin it until they have seen enough sustained effort and success around them to make it possible for them to believe that this is something more than a passing fad. There will also be a range of views among parents. Some parents will encourage change, others will worry that the schools are experimenting on their children. In each school or school system, it will be necessary to make some visible changes and make clear the benefits for students in order to persuade the skeptics. It will be necessary, that is, to begin the process of change while continuing to try to develop support for change.

Vignette: Individuals can initiate change

During 1993, the Monatiquot School in Braintree, adopted the NCTM standards as a way of revitalizing mathematics teaching in grades 3-5. Louise Moline, the principal, encourages and supports professional development. When she received information about the Massachusetts Academy for Teachers (a program offered through the Massachusetts Field Center for Teaching and Learning) she passed it along to a teacher she thought might be interested. This is her practice. Rather than merely putting publications in the teachers' room, she makes a personal approach to teachers who have expressed interest in a particular topic. In this instance, she passed the Academy information to Dave Babel, a fourth-grade teacher who was a member of the town-wide math committee.

Mr. Babel applied to the Academy and was accepted. He attended a two-week summer workshop and follow-up meetings on several Saturdays throughout the year. He received a stipend from the Academy for participating. From the Academy, he learned about the NCTM Standards. He discovered that while other school systems were working toward implementing them, Braintree had not begun this process.

Mrs. Moline asked Mr. Babel to present what he had learned at two half-day teacher workshops. The teachers at Monatiquot were receptive to broadening the mathematics content they were teaching, and decided to try using the NCTM recommendations in their classrooms. Mrs. Moline encouraged their attempts and allocated time at monthly faculty meetings so teachers could share information about what they were doing. The teachers brought mathematics activities they had tried with their students and received feedback from colleagues. This collaboration and discussion of change led the teachers of Monatiquot to adopt a problem-solving approach to teaching mathematics as one of their school-wide goals for the following year.

In the fall of 1993, Mr. Babel applied for and received a Commonwealth Inservice Institute grant to contract an outside teaching team to offer a course called "Implementing the NCTM Standards in Third, Fourth, and Fifth Grade Classrooms." The course was offered to Chapter I teachers as well as all third, fourth, and fifth grade teachers. Teachers could attend at no charge and receive three Braintree increment credits, or they could pay $140 and receive college credits from Fitchburg State College. The course was held after school at the Monatiquot School for ten three-hour sessions in the fall. The relevant content of the course, the personalities of the instructors, the chance to earn free or low-cost credits at a convenient location, and plenty of refreshments all made the long hours bearable!

What started as one brochure sent to a principal who gave it to an interested teacher ended as the beginning of town-wide change. The essential elements of the process included the encouragement of the principal, outside funding for Academy participants, sharing among teachers at faculty meetings, and the Commonwealth Inservice Institute grant. One principal and one teacher were able to influence many others..

–Deborah King

Time

Systemic change takes sustained effort over time, and this makes the process more complex.

As teachers and administrators are persuaded of the possibility of significant improvement in the teaching of mathematics and science, and are persuaded that the changes being recommended represent improvement, they will need time to reconstruct these ideas for themselves and adapt them to their schools, classrooms, and students. This part of the process will require more than a year or two.

In addition, teachers and administrators will need to meet together to rewrite curriculum guidelines, to redesign courses, to select appropriate teaching materials, and to revise methods of assessing students and programs. They will not be prepared to do this well until they have made some beginning changes in their teaching and the curriculum. These programmatic changes, therefore, can not be made effectively within the first or second year.

Planning to support a sustained effort

A long-range plan which is developed in consultation with all segments of the school community, and which is well publicized and monitored, will be essential for at least two reasons.

First, a long-range plan will help everyone understand that some parts of the process build on other parts—it would be foolish, for example, to invest in expensive materials, technology, or textbook series before teachers and administrators have enough experience with these new ideas and teaching methods to make appropriate choices. A timetable and periodic monitoring will provide assurance that progress is being made when the process goes more slowly than some might wish or expect.

Second, a long-range plan will help maintain a focus in the midst of competing needs. Before the new curricula in mathematics and science are fully implemented, or before the implementation is even well begun, there will be other pressing needs in the school system. Social studies, English, and art, for example, can not simply be ignored for four or five years as people work only on mathematics and science. In addition, as teachers and administrators well know, there are many non-curricular topics which will require attention—violence prevention, AIDS, or increasing numbers of students of immigrant families, for

example. It is the nature of change in society that some of the important topics for the next four or five years are probably not yet known.

It will be necessary for schools and/or school systems to develop long-range plans which will enable them to provide the sustained attention required to make the changes in the teaching of mathematics and science and, at the same time, support review and change in other areas of the curriculum and respond to non-curricular or unanticipated issues and needs. For, as everyone now understands, the improvements needed in our schools will not be achieved without sustained attention.

Communication

Implementation of these frameworks will also require effective communication. A communication plan must ensure articulation among the various groups (administration, teachers, students, parents, and community), and among teachers and schools, both within the district and within the state. Why reinvent the wheel if another school or district has already created it?

Planning and evaluating progress is a cyclical process

The chart on the following page illustrates some aspects of planning which will need to be considered, and also illustrates the recursive nature of the processes of assessing/planning/acting/assessing. A blank chart is included in Appendix 3.

Determine Existing Local Conditions and Potential Resources	Establish Goals and Create an Action Plan to Meet Them	Continuously Evaluate Progress on Aspects of the Plan
• Begin with an assessment of strengths, resources, aspirations, and needs. • Identify potential committee members and leaders for the implementation process. (Invite administrators, teachers, parents, and community members to volunteer and to identify others who have a contribution to make.) • Determine the skills and expertise of administrators, teachers, parents, and community members. (Again, encourage both self-identification and identification of others.) • Determine the perceptions of administrators, teachers, and parents regarding the current programs and hopes for new programs (i.e. polls, questionnaires, and/or focus groups). • Inventory materials, including technology. • Survey physical plant facilities. • Contact nearby school districts, resources within the Commonwealth, and national centers. Steal their good ideas. It isn't necessary for each school district to invent an entirely new plan.	• Arrange for groups of people to brainstorm, "We could do this ... or that." • Establish both long-term and short-term goals. • Include communication as part of the plan. • Keep everyone informed of the long-term plan and progress in carrying it out. • Develop a professional development program to support implementation of the frameworks. • Select and create school and/or district curricula for mathematics and science. • Prioritize needed materials and equipment and develop a timeline for securing them.	• Assess the effectiveness of the professional development program. • Assess the effectiveness of communication. • Assess the extent to which teachers, administrators, parents, community people take leadership. • Assess the implementation of the new curricula. • Assess the teaching of the new curricula. • Assess student learning. • Assess the involvement of teachers, administrators, parents, and community. • Quantify what can be quantified. • Include systematic collection and analysis of non-quantifiable data as well.
Continue to assess conditions and progress, and revise the plan.		

Systemic Reform within School Systems

As described earlier, developing effective mathematics and science programs for all students within each school district will require thoughtful, sustained attention all aspects of the system which affect the quality of the educational program. This will include attention to professional development, curriculum review and modification, selection of instructional materials, assessment of students and programs, and school and district policies.

Professional Development

Effective math and science programs depend on teachers and administrators who are enthusiastic, informed, creative, dedicated, provided with adequate resources, and supported to develop a productive professional culture within their schools. Teaching requires ongoing professional development, and teachers need to experience the kind of instruction that they are being asked to provide. Administrators need ongoing professional development in order to support these changes. In addition, as teachers and administrators make these changes, they need to deepen their understanding of science and mathematics, in order to avoid making changes which are not well thought out. Creating professional development programs which support orderly change in school districts, promote a culture of inquiry, and meet the needs of individual teachers and administrators, will be a challenge—although it need not be an insurmountable one. The following section presents suggested guidelines for the planning process and for the activities and opportunities offered to teachers and administrators.

Teachers are learners. One of the tenets of the Frameworks is that teachers are most effective in designing curriculum and instructional strategies when they make explicit and build their work on the assumption that all students can learn. Similarly, professional development programs will be most effective if their design is consistent with the assumption that teachers want to be successful in their teaching, that they will welcome well-designed, appropriate opportunities for learning which will enhance their teaching. This means placing more emphasis on supporting teachers' growth and using their good ideas than on attempting to use professional development activities to correct poor teaching. Professional development can support those teachers who take leadership within the district. Inservice programs are sometimes seen as a way of enforcing standards for teaching. This does not work well and it inevitably distorts inservice programs.

Be clear about long-term purposes and expect to develop the program over time. It will be important to establish clearly that encouraging and supporting leadership on the part of teachers and principals is one of the long-term goals of the program. It may well be, however, that few participants respond initially to an invitation to lead workshops for their peers, or take other leadership positions. Some people who have a contribution to make won't volunteer; they won't realize that what they are doing is in any way special, and will

need to be invited. Others won't agree to lead until it is somewhat established as a norm within the district. Again, it will take time to develop a culture within the district that supports teachers and principals working effectively to promote adult learning.

Include principals and other administrators. Administrators also need opportunities to learn. If the goals of these frameworks are to be achieved, principals will need to understand the changes teachers are making, develop new images of "good teaching," and communicate effectively with parents about the new programs. They will need opportunities for sustained discussions with teachers, and with other administrators.

Reward initiative and risk-taking. As teachers and administrators study and plan together, they will develop good ideas which they will be unable to implement without support. Administrators need to advocate for teachers, and to help teachers to advocate for themselves. Sometimes a school staff will need money, or assistance in proposal writing. Teachers will often need administrative flexibility, such as modifications in the schedule. Similarly, as teachers take more initiative and take risks, they will sometimes try things which do not work and they will need support at those times.

Teachers have diverse needs. Again, one of the tenets of these Frameworks is that students as learners differ from one another in a variety of ways. Professional development programs will need to take into account that teachers' needs and interests will differ from teacher to teacher and will change over time. As suggested earlier, there will need to be a variety of forms of professional development. Inservice programs will need to be strengthened and other forms of professional development developed. At some points, for some teachers and principals, a lecture will be an appropriate method of conveying information. For much of the work, however, other forms will be more appropriate.

Offer direct hands-on mathematics and science experiences. What teachers are being asked to do is not procedural. It can not be learned by listening to a careful explanation. Teachers will need to try out new ideas. Their understanding of the need for and the nature of new programs will not occur just by hearing about them or even by trying to implement them. As teachers participate in hands-on experiences, they will also directly experience a variety of instructional strategies such as working together in small groups, using

manipulatives, using appropriate technology, and writing about mathematics and science. These experiences will enhance teachers' own understandings of mathematics and science and, at the same time, provide new models for their own teaching.

Provide time and opportunities to reflect on and grapple with issues. Teachers need opportunities to come together and discuss, write about, and reflect upon educational issues and their own learning and teaching experiences. They need opportunities to confront and examine issues and their assumptions and beliefs about mathematics and science and learning. Teachers need opportunities to read articles, observe students, observe each other, and discuss their reactions. This process of grappling with issues is too rarely a part of professional development, but is critical for sustaining the change process, and hence for lasting change to occur.

Address assessment as well as instruction. Pre-service and inservice education in assessment practices will be vital in order to implement new assessment strategies. It has been found that pre-service courses in evaluation and measurement (when taken) often fail to address the day to day concerns of teachers. Most courses emphasize using standardized tests, developing paper and pencil tests, and statistical aspects of assessment, while virtually ignoring daily assignments and oral discussions, performance and portfolio assessment, critiquing of company published tests, and reviewing key strategies of assessment.

Even after participating in illuminating pre-service and inservice instruction, teachers must go back to the realities of their individual classrooms and apply the concepts and strategies that they have learned. Assessment reform will entail an ongoing conversation among teachers, so that they can share their attempts to integrate assessment, subject content, and pedagogical style. Time needs to be allotted for teachers to collaborate with colleagues on developing appropriate, authentic assessment items that will be integrated into the curricula that they are teaching. This need for collegial communication is supported by research that has found that teachers currently rely more on colleagues than they do on prior measurement and evaluation courses for the development of their assessment schemes.

Principals, as well as teachers, will need to balance their need to plan ahead with the realization that classroom assessment provides

insights and information that will alter upcoming instruction. Teachers will require the ongoing interest and support of principals to create a school community where assessment is an ongoing topic of consideration and where risk taking with alternative assessments is encouraged.

Encourage collaboration. Teachers and administrators need opportunities to collaborate with one another—to share ideas, give each other encouragement, plan lessons together, share what worked and what didn't work, share materials and resources and build upon each other's strengths. Collaborative professional relationships are an essential ingredient in bringing about changes in mathematics and science instruction, and they must be fostered and supported by school systems so that teachers have the time and a structure for doing this.

Promote communication within and between grade levels, departments, and schools. To establish a well-defined, complete program, it is helpful to include some portion of the training with heterogeneous groups of teachers across grade levels. It is important for elementary teachers to see where the curriculum is going and for secondary teachers to understand where students have been.

Establish a diverse planning committee, which is clear about its authority and responsibility. Developing a professional development program which has the characteristics discussed above will require long-rang planning. It will be important that one committee be responsible for planning the ongoing process. The system will benefit if the planning committee has authority and responsibility for all aspects of professional development, not only for the inservice program.

Designing sessions and managing the schedule for an inservice program which is responsive to the needs of all the faculty is an important and demanding task. It can best be done by a committee which reflects a variety of these groups.

In addition, an effective professional development program needs to include more than the inservice program for the district. Teachers and administrators need opportunities to attend conferences and visit other schools. They should be encouraged to prepare and carry out other activities which will support their continuing professional development. Local universities should be approached to determine the feasibility of on-site courses for credit. Teacher-led study groups

should be supported where they are initiated by teachers. These activities may also enrich the inservice program. Teachers who have attended a conference and/or made visits to other schools may usefully share their experiences at a later inservice program.

Provide adequate human and other resources. It will take substantial time to make long-range plans, organize and coordinate small groups, provide some choice for participants, design a schedule that allows groups to meet more than once, and establish guidelines for opportunities which go beyond inservice programs. This type of program takes more planning than one which consists of lectures by outside experts. It also requires flexibility in the use of funds and in use of time, including flexibility in use of professional days and duties.

[Reader: We would like to include a vignette of an exemplary professional development program.]

Curriculum Review and Modification

One of the essential continuing processes within each school district will be the review and modification of the curriculum. What will be required in order to establish curriculum based on these framework?

It will be essential that the process include time for teachers and administrators to review the current curriculum, K-12. This will entail time for review and discussion of official written documents, discussion of how the documents are interpreted and enacted by teachers throughout the school district, and an evaluation of the current curriculum. Time should be made available for discussions of the achievement of students K-12, of their successes, progress, and difficulties. As noted earlier, it will be important that teachers and administrators have time to learn about the changes which are being recommended, and that they have time to test some of the ideas in their own classrooms and schools, before they begin formally to redesign the curriculum. It will also be important that arrangements be made for teachers and administrators to have access to the work of others—good ideas from other schools in Massachusetts and from national projects. Teachers and administrators will want to find out what technology is available, and what will be most useful in their schools.

Teachers and administrators will need to work together to establish a plan for designing and implementing a new curriculum—

including a plan for involving and communicating with parents. It will be important to include a time line and a plan for ongoing communication and coordination K-12. In addition, the curriculum should include a plan for program assessment and student assessment.

Selection of Instructional Materials

The instructional materials selected in schools and school systems will make concrete the goals and ideas of the mathematics and science curricula. They will provide a foundation for instruction, often determining to a great extent the subject matter that students encounter. They will influence what and how teachers teach and what and how students learn. Good materials can significantly improve students' attitudes and achievement. The selection of materials, therefore, should be preceded by extensive planning, long-term professional development, and piloting of materials. The instructional materials must be consistent with the goals and objectives of these Frameworks and of the district's curriculum. Careful attention should be given to selection of all resources that assist in the instruction process, including, for example:

- textbooks and ancillary materials,
- other printed materials,
- manipulative materials,
- computer software,
- video materials (tape and laser discs),
- distance education programs,
- teacher training programs,
- programs that result from new technologies, and
- student assessment materials.

It will be important that local school districts appoint committees to select instructional materials and, further, that these committees are representative of interested parties, including parents, businesses, and higher education groups. The Association of State Supervisors of Mathematics and the National Council of Supervisors of Mathematics have published a very useful *Guide to Selecting Instructional Materials for Mathematics Education* (1993). Although some recommendations are subject specific, many, such as those related to establishing a committee, determining criteria to be used, and evaluating materials apply to any subject area. In addition, selection committees may find it useful to consider the following questions

developed by the PALMS Specialists in the fall of 1992.

- Does the unit contain significant mathematics/ science content, building toward a deeper understanding of mathematics/science concepts?
- Do the activities engage learners in authentic tasks of inquiry, reasoning, and problem solving that reflect real-world scientific and mathematical practice?
- Do the activities encourage the learner to generate questions, make decisions, and conduct investigations?
- Does the unit allow for collaborative interaction of learners?
- Do the activities promote effective communication?
- Does the curriculum take into account the varied learning styles and cognitive/affective/physical needs of learners in the class-room?
- Does the unit encourage inter/intra connections between math/ science and other curricula areas?
- Does the technology, when used, effectively promote the PALMS Philosophy?
- Does the curriculum reflect a variety of cultural perspectives?
- Does the curriculum utilize approaches and materials that are unbiased (diverse) with regard to ability, race, gender, economic, and cultural backgrounds?
- Does the curriculum include a variety of ongoing assessments designed to improve instruction and enhance learning?
- Are the reading levels of the instructional materials appropriate?

Examining Issues of Equity

As discussed in Chapter 1, one of the central equity issues is how students are grouped for instruction. In common with schools throughout the U.S., many schools in Massachusetts group students on the basis of performance or perceived ability. The assumption underpinning these practices is that teachers are better able to deliver what individual students need if the differences among individuals in a classroom group are not too great. Theoretically, teachers are able to provide more appropriate assistance to students in classes that are more homogeneous. Grouping is, therefore, designed to support and enhance the learning of all students.

Given the recent research showing that tracking often leads to inequities and decreased learning, some schools are now moving away from tracking students into separate classrooms. These school systems have already started, and will continue, to move toward

more flexible grouping and more heterogeneous grouping. This is a complex task. Eliminating tracking and taking no other action will not ensure increased learning. Announcing that students will be placed in more heterogeneous groups will not create better learning environments and may simply create different problems. Renaming courses will not help. Although the negative effects of tracking are now well documented, effective alternatives, and plans for implementing them, are not yet widely tested or available. Administrators and teachers will need to work together to change the curriculum and teaching methods so they are appropriate for more heterogeneous groups. It may be important to decrease class size. Teachers will need access to resources, and time to try out new instructional materials which are appropriate for more heterogeneous groups of students. They will need to meet together to develop new grading procedures.

Schools and school systems will find that some equity-related program outcomes will be relatively easy to monitor, but that others will require more careful study. Some questions that will be relatively easy to monitor include the following:

- Are there disproportionately fewer girls or students of color in advanced and AP classes?
- What are the average grades for girls and students of color? How do they compare with the average grades for the school?
- Is the same curriculum content and the same quality of teaching available to all students even if not always at the same pace?
- Are opportunities for extra help or enrichment available for all students?

Questions that will require more careful study include:

- What attitudes toward mathematics, science, and school are students developing? Do they see mathematics and science as interesting? As potentially useful in their lives?
- What are the attitudes toward school of those students who perform least well?
- Are students developing and maintaining persistence in learning? Do they become more willing/able as they get older to keep trying when working on a difficult task?
- Do students believe that learning is a lifelong process?
- Are students locked into sequences of courses? Is it possible to change? How often does it happen?

Massachusetts State Department regulations require that each school district establish policies that will insure the removal of all

obstacles that may impede equal access to educational programs for students, regardless of race, color, sex, religion or national origin. In addition, school committees and superintendents are required to provide all school personnel with the information and inservice training they need to advance the educational goals of the school in a manner free from discrimination.

[Reader: We would like to include a vignette about a school or district that is examining issues of equity.]

Conclusions and directions for the future

The issues discussed in this chapter indicate the scope of the effort involved in implementing the Mathematics and the Science and Technology frameworks in the Commonwealth of Massachusetts. As we look toward the future, we recognize the need to monitor the progress of this effort, which many districts may find will require more attention, time, and funding than has been devoted to such activities than in the past. Although the goals and methods of monitoring frameworks implementation will vary by community, the following four questions are suggested as guidelines.

To what extent is the school/district progressing in:

- aligning its mathematics and science instructional programs with the frameworks? Are students learning important mathematics and science content, and are they learning it in ways consistent with instructional practices described in the frameworks?
- involving teachers and parents in designing, implementing, and assessing new programs?
- collecting, analyzing, and reflecting upon appropriate information, both qualitative and quantitative, to inform decisions?
- providing information about school programs to the community and other partners on an ongoing basis?

Providing mathematics and science programs which support the learning of all students will require that schools and school systems specify goals for mathematics and science programs, monitor and assess local performance in relation to those goals, and confront numerous difficult and complex educational issues. As with all other aspects of this effort, it will be important to engage all stakeholders in the partnership for frameworks implementation, and search widely for financial support and other relevant resources.

This Framework closes with vignettes that illustrate the enthusiasm of two teachers as they strive to improve learning for their

students. Like them, there are many other teachers, schools, and districts in Massachusetts who are already working diligently to bring about the changes called for in these Frameworks. They know that the effort is arduous and that as soon as one task is accomplished, another one looms ahead. They also know that as they work toward their goals, they are rewarded daily—when they see their students become excited about learning mathematics and science and when their students are able to use mathematics and science concepts to investigate and solve complex problems. Achieving the goal of mathematical power and scientific literacy for all students will be well worth the effort.

Mathematics Vignette: Collaborating to untrack and integrate secondary mathematics

Untracking and integrating the secondary mathematics curriculum takes a lot of work, but can be done. Our school is doing it. In some respects, Pioneer Valley Regional School is ahead of the times. We began a process of untracking the entire school more than ten years ago.

Pioneer Valley is a regional school for four rural communities. About 460 students in grades 7-12 attend the school. In 1985, after an extensive self-study involving teachers and students, the School Committee adopted a policy of heterogeneous grouping of students for all courses. To support the program changes, teachers were provided staff development related to teaching styles, teaching strategies, and holding high expectations for students. Teachers were asked to try suggested strategies in their classrooms and report back what happened. The social studies and the English department jumped into the process wholeheartedly; the mathematics and science departments ventured more slowly.

I came to Pioneer Valley in 1987 as the seventh grade math teacher. I had fifteen years experience teaching grades 4, 5, and 6. I was used to working with heterogeneous classes and having more than one thing going on at the same time. I rewrote the seventh grade mathematics curriculum as I taught it. Other teachers became curious about what I was doing. They were frustrated trying to teach students in a heterogeneous class, and came into my class to observe. I didn't want to push them, but I wanted them to see that it could work.

The next year I taught one eighth grade class and two other teachers taught the other classes. We were all dissatisfied with the curriculum. We decided to work together to change it, meeting once every few weeks to jointly plan. One decision we made was not to follow the order of the textbook. For example, we decided to teach perimeter and area while we were teaching operations on whole numbers and decimals because we thought the topics should be integrated. We spent a lot of time discussing classroom management and instructional strategies. We tried things we had not done before, and shared with each other our successes and failures. We finished the year agreeing that the textbooks were not challenging enough and decided to purchase a new text that included the use of variables and more engaging and challenging explorations for the students.

The entire mathematics department (grades 7-12) selected the new texts since we knew the selection would impact everyone. At the selection meeting, several algebra and geometry teachers shared some of their concerns about teaching heterogeneous classes. There were many students who were not making connections with the mathematics and didn't seem to remember much. Some students were having so much difficulty that it was hard to get them to participate at all in class. The teachers wondered whether more students would be successful if the teachers could teach algebra and geometry at the same time. We started looking for an integrated textbook, but at that time (1988) found nothing, so we kept the standard courses. However, we continued to meet together thinking about and discussing the high school curriculum. Students gave us feedback that when they took the college boards, they felt that they didn't have enough geometry. We decided to offer the option that students could take either algebra 2 or geometry following algebra 1.

As a result of the staff development program, teachers made a number of changes in their classes. Students did more group work. Teachers conveyed to students high expectations. When the students saw that the teachers really believed that the students could be successful in mathematics, the enrollment in the ninth grade general math course

dropped to six students. Those students were incorporated into the algebra class, and they were successful. Yes, some students struggle more than others, but a teacher doesn't have to lower his or her expectations in a heterogeneous class.

During the 1991-92 school year, we found three new integrated texts. We reviewed them carefully, not only considering the topics that were included (such as the incorporation of matrices and trigonometry ideas), but also that flexibility was built into the texts so they could be used effectively in heterogeneous classes. Questions we considered included: Did the texts include a variety of examples and applications at many different levels so students could proceed from simple to more complex problem solving situations? Were algebra and geometry truly integrated rather than presented alternately within the texts?

We are phasing in the new, integrated program a year at a time. In 1992-93, all ninth graders were enrolled; in 1993-94, all ninth and tenth graders will take integrated mathematics; and during the following year, the program will extend to eleventh graders.

In all our mathematics classes at Pioneer Valley, we are using a variety of approaches to explore mathematical ideas. For example, when we are working on the Pythagorean theorem, students do a hands-on proof, actually building squares on the legs of right triangles. They also solve the theorem algebraically and using a geometric proof. Teachers present for about five minutes at the beginning of a period. Students work on problems both in cooperative groups and alone. Often they work with a partner, and then share results in a foursome. Students are asked to write and explain their thinking. At times, a group product is required.

Assessment is often rather unorthodox. For example, last year I asked students to make a design that used several different figures, compute the area of each figure, and record how they tackled the problem. Prior to working on the project, the students contributed to designing how it would be assessed, deciding how much each part of the product would be worth. It took about a week for me to review the sixty projects, but I gained many insights about my students' understandings. When I couldn't understand their work, I asked students to come up and explain their procedure to me.

Untracking and integrating the secondary mathematics program at Pioneer Valley has been a long process, but at the same time, rewarding. The program we have implemented is one that also can work at other schools. For example, we have visited Central Park East High School in Harlem, and they have visited us. We have found that both schools are using similar kinds of strategies, and that they work equally well both places.

What has made our efforts at Pioneer Valley successful, is a history of open communication. As teachers, we have a willingness to observe and be observed by one another. We discuss individual students among ourselves and with others, such as resource teachers, who work with them. Collaborating with one another has not always been easy. We have found that our efforts have resulted in an improved mathematics program for our students, and also, that we have grown professionally as teachers. Teaching is never dull for us; we are excited as we share with one another the evidence we see in our classrooms that our students are making sense of mathematics.

Susan Currier, Pioneer Valley Regional School, Northfield

Science Vignette: A passion for students, teaching, and the sea

"There are more people alive today than have ever died." I wrote the sentence on the board that I had heard Robert Ballard say last night. Dr. Ballard, the Woods Hole scientist who found the Titanic, the Lusitania, and the Bismarck and is pioneering robotics in undersea exploration, was the speaker at a dinner meeting I attended. My eighth grade students came into the class, sat down, read the statement on the board, and immediately began talking among themselves. After about five minutes of active dialogue, one student asked, "What are we supposed to do today?" "You're doing it," I replied. "You're reacting to this statement the same way I reacted. What are the implications for us?"

I teach science, or more specifically, ocean sciences, to the 125 eighth grade students enrolled in a middle school in Cape Cod. The inception of the program, fifteen years ago, was a combination of a love I have for the ocean, and the frustration I felt for the general science program I was teaching. I believed I could teach the same science better and make it more relevant using local applications. I scrapped a botany program that was based on corn, and a zoology program on perch dissection. I convinced the principal to let me try out the ocean centered approach.

My classroom was equipped with two microscopes, and I needed 25! The principal agreed to buy two a year, but finally upped the amount to six per year. I begged and borrowed other equipment, sending a wish list to parents, the local museums, and the newspaper want-ads. In response, the class received fish tanks, air pumps, general aquarium supplies, and five refrigerators. The local museum of natural history gave me a catalog of free resources, and I spent evenings writing about 75 letters requesting curriculum materials.

My major focus was on getting the students involved doing science. I found a fantastic book, Experiments in Oceanography, that had suggestions for hands-on activities plus how to build the needed tools, for example, how to build a plankton net out of panty-hose.

Throughout the years the course has evolved to a routine schedule, but one that I hope is engaging and challenging for the students. "Creature Feature" happens every Monday. I bring in 10-15 specimens of a creature, perhaps star fish or horseshoe crabs, and students are to look at the creature, and say as much as they can about what they observe. At first this is difficult for the students. They say things like it has claws or a shell. But the more they do this, the better they become in identifying distinguishing characteristics of animals.

On Fridays we do experiments in physical oceanography, examining nonliving aspects of the ocean, such as waves, tides, and salinity. One way we have investigated currents is by repeating Benjamin Franklin's experiment and inserting cards in bottles and dropping them into the ocean. For more than ten years, on March 20, the Coast Guard has taken students 17 miles out to the same buoy in shipping lanes. Of the 2000 bottles that have been set adrift, about 30 have been returned from such diverse locations as Ireland, Scotland, Portugal, France, Sweden, Bermuda, Nova Scotia, and Ipswich.

Reading, writing, mathematics, and history are integrated into the course. Each spring students collect data on herring migration, when herring, in a way somewhat similar to salmon, return to their birthplace to lay eggs. (Unlike salmon, herring don't die at the end of this process.) Students record the weight, length, and sex for a sample of herring, find the average for the data, and compare their findings with those recorded for previous years. One interest-

ing finding has been that following the blizzard of 1978, the 1979 herring catch had an average weight drop and a length increase! Students conjecture why this might have occurred since that during the blizzard the herring were many miles south of New England. Students also learn about one of the major commercial uses of herrings as fertilizer, and that Indians also used herrings for fertilizer and this saved the lives of many Pilgrims.

The course is rigorous and demanding for the students. While eighth graders vigorously protest structure, internally they want it. Students receive a monthly learning agenda that specifies all assignments, including homework. I tell them that they will get no surprise quizzes, no weekend homework, and in exchange, there are no excuses. Work must be done on time. For some assignments, particularly lab ones, students work collaboratively. Other assignments are done independently. My overall emphasis, however, is to treat this program as an attitude not a course—we are interrelated to the ocean, and the ocean profoundly affects our lives.

When we moved into a new middle school, it was decided that each grade level choose a name and a theme. I suggested "Stewardship"—building the sense of responsibility for the earth. A ship's wheel became the logo, with the spokes representing the different disciplines, helping to steer the students through the sea of life. Students learn, by engaging in a variety of activities in each subject, that stewardship centers around being proactive, rather than reactive or inactive. For example, eighth graders help clean up garbage on the beach on Saturdays, not because they left it there or simply to help make the beach more attractive, but because of the belief that even though this is not our garbage, we are all going to be poisoned by it.

I want to instill in my students that each one of them can make a difference. In 1976 one of my students wrote a letter to our congressman, that was recorded in the congressional letter, stating how precious Stellwagen Bank, an underwater shelf used as a habitat by local whale populations, is and recommending that it be declared a sanctuary. In 1992 that actually happened. My former student, Richard Comeau, is now an officer in the merchant marine.

All human beings need a passion in their lives. I have found mine. When my students see there is a purpose to what they do in school, when they make connections between school and the world, their eyes light up, and I hope that maybe, they will get a sense of the excitement that can develop when one is committed to making a difference in events that affect the quality of all our lives.

George Kurlychek, Harwich Middle School

Appendix 1
Regulations of Special Relevance to Science Educators

Some regulations that are of particular significance to science educators are listed below. These regulations pertain to protective eye devices, treatment of animals, and hazardous substances.

General Laws chapter 71, 55C. Eye Protection Devices

Each teacher and pupil of any school, public or private, shall, while attending school classes in industrial art or vocational shops or laboratories in which caustic or explosive chemicals, hot liquids or solids, hot molten metals, or explosives are used or in which welding of any type, repair or servicing of vehicles, heat treatment or tempering of metals, or the milling, sawing, stamping or cutting of solid materials, or any similar dangerous process is taught, exposure to which may be a source of danger to the eyes, wear an industrial quality eye protective device, approved by the department of public safety. Each visitor to any such classroom or laboratory shall also be required to wear such protective device.

General Laws chapter 272, 80 G. Experiments of Vertebrates; vivisection, dissection of animals; care

No school principal, administrator or teacher shall allow any live vertebrate to be used in any elementary or high school under state control or supported wholly or partly by public money of the state as part of a scientific experiment or for any other purpose in which said vertebrates are experimentally medicated or drugged in a manner to cause painful reactions or to induce painful or lethal pathological condition, or in which said vertebrates are injured through any other type of treatment, experiment or procedure including but not limited to anesthetization or electric shock, or where the normal health of said animal is interfered with or where pain or distress is caused.

No person shall, in the presence of a pupil in any elementary or high school under state control or supported wholly or partly by public money of the state, practice vivisection, or exhibit a vivisected animal. Dissection of dead animals or any portions thereof in such

schools shall be confined to the class room and to the presence of pupils engaged in the study to be promoted hereby, and shall in no case be for the purpose of exhibition.

Live animals used as class pets or for purposed not prohibited in paragraphs one and two hereof in such schools shall be housed or cared for in a safe and humane manner. Said animals shall not remain in school over periods when such schools are not in session, unless adequate care is provided at all times.

The provisions of the preceding three paragraphs shall also apply to any activity associated with or sponsored by the school.

Whoever violates the provisions of this section shall be punished by a fine of not more than one hundred dollars.

Hazardous Substances

School districts also need to comply with regulations having to do with hazardous substances. Among important regulations for school districts to know about are:

General Laws chapter111F, Hazardous Substances Disclosure by Employers which deals with the disclosure of hazardous substances by employers, and

Hazardous Waste Regulations 310 CMR 30.000 which addresses the disposal of hazardous waste.

Appendix 2: Clarifying Roles in Systemic Change

The lists which follow are suggested as a basis for discussions to define and clarify the contributions each group might make to this important effort.

Teachers

Teachers are key figures in changing the way mathematics and science are taught and learned. As discussed in greater detail below, opportunities for professional development must be made available to all teachers if the Frameworks are to be implemented successfully. Making professional development a part of the school day and hiring teachers as leaders in mathematics and science will help make the Frameworks' vision a reality. In addition, teachers need to take increased ownership for their own development as professional educators.

Appropriate role activities for teachers can be considered within three broad categories.

Professional development

- Improve their own mathematics and science knowledge.
- Seek out opportunities for professional growth.
- Attend workshops and conferences.
- Join professional organizations.
- Continue to be lifelong learners.
- Read research and other professional publications.
- Participate in workshops and courses that cover content and pedagogy.

Teaching

- Hold high expectations for all their students.
- Notice their students strengths and build upon them.
- Elicit their students interests, and incorporate them into the curriculum.
- Make use of hands-on experiences and a variety of other instructional techniques.
- Meet with other teachers to reflect upon their own teaching and learning.
- Reflect upon educational issues and experiences.
- Try new instructional approaches.
- Plan lessons suitable for the diversity of students in the classroom.

Participation in and contribution to the school community

- Demonstrate leadership in their own schools and/or districts.
- Participate in school-based instructional support teams.
- Form study groups within schools.
- Model good teaching and learning.
- Join school-based committees.
- Collaborate with colleagues.
- Help make positive changes.
- Apply for grants to support needed programs.

Principals

Schools in which principals work with staff to adopt a plan of action that allows for experimentation and risk-taking make the implementation of new curricula much easier. Principals can support teachers who take on new roles and responsibilities by creating environments conducive to change. The school is an appropriate target area for changes in education, and it is the principal who is the key element for adopting and using new practices in a school.

Appropriate role activities for principals can be considered within three broad categories.

Professional development

- Become familiar with the Massachusetts Education Reform Act and the Curriculum Frameworks.
- Be aware of the current research on effective staff development.
- Support and participate in professional development activities.
- Keep current on district-adopted curriculum materials.
- Become knowledgeable about national standards and recognize the components of the Frameworks.
- Participate in the professional development process at the district and state levels.

Attitudes and behaviors in the school

- Encourage a positive, supportive climate that is conducive to change and teacher growth.
- Recognize teachers for professional work.
- Write letters of commendations for teachers who take on extra leadership responsibilities.
- Place a priority on curriculum issues.
- Communicate and consult with teachers.
- Motivate staff and encourage them to take risks.

- Discuss the difficulties of change with staff: time /pain/etc.
- Supervise and evaluate staff using multiple strategies for improving instruction.
- Be a catalyst for reflective practice.
- Create a school-wide plan with teachers for professional development.
- Implement peer-coaching in the school as a tool for improving instruction.
- Report mathematics and science activities and success stories to the media.
- Sponsor Family Math and/or Family Science Nights.
- Be visible in the school — visit classrooms, teach.

Material support for teaching and change

- Identify teacher leaders who can support other teachers.
- Set aside money for substitute teachers so teachers can attend conferences or visit other schools.
- Fund professional activities such as workshops and conferences.
- Offer release time to teachers to complete professional activities.
- Establish incentives for teachers who use new instructional techniques.
- Provide materials necessary for change.
- Make staff development part of the teachers' work role.
- Identify local resources (mentors, specialists, college faculty, parents, people from businesses, etc.) who can contribute to professional development programs.
- Support professional development in terms of time, money, resources, and materials.
- Use a variety of resources for the evaluation of staff.
- Apply for additional grant money to support needed programs.

Parents

As discussed in Chapter 4, parents are important resources for schools. Appropriate activities include the following.

- Be learners with their children.
- Participate in educational endeavors.
- Attend family/parent math and science programs at school.
- Demand equity of funds.
- Take responsibility for their children's education.

Students

Ultimately, the primary beneficiaries of improved mathematics and science teaching are the students in our schools. They, too, have an essential contribution to make.

- Study hard to learn and master skills
- Take responsibility for their own learning
- Reflect on what they are learning.

District level administrators and staff

The school district has the responsibility of ensuring that the goals for professional development stated in the Massachusetts Education Reform Act and the Massachusetts Curriculum Frameworks are met. The district therefore plays a major role in providing the resources necessary for the professional development of educational staff.

Appropriate role activities for district administrators and staff can be considered within three broad categories.

District policies and structure

- Restructure the school day to accommodate professional development activities so they don't remain as add-ons.
- Reward and recognize teachers who assume leadership roles and responsibilities.
- Hold principals and teachers accountable for ongoing professional development and lifelong learning. Link individual professional development plans for district math and science teachers to the district and state goals of implementing these frameworks.
- Revise assessment measures so that they are aligned with the proposed change.
- Reexamine the teacher evaluation process.
- Establish outreach to the community to support quality mathematics and science programs.

Material resources

- Provide time to staff to participate in professional development opportunities.
- Improve and continue to provide in-service programming.
- Create a professional development resource center from which teachers and principals may borrow materials.
- Funnel more money to schools to support professional development action plans.
- Establish professional development schools.
- Supply resources that support in-service programs.

- Create partnerships with cultural institutions, businesses, and universities.
- Provide in-service to support the implementation of the Massachusetts Curriculum Frameworks.
- Allow for teacher sabbaticals.
- Work to optimize class sizes.

Human resources

- Hire mathematics and science specialists who can make classroom visits and offer support in terms of ideas, time, resources, etc.
- Help schools identify needs and find ways to meet those needs.
- Implement mentor programs for beginning teachers.
- Hire quality mathematics and science teachers.

School Committees

Real change and improvement in mathematics and science education require sustained thought and action by the entire community. This process will be guided and supported by school committees throughout the commonwealth. Appropriate activities include the following.

- Study the frameworks and standards.
- Hire superintendents and other administrators who will provide instructional leadership for implementation.
- Provide funding for sound programs.
- Assure that the district provides meaningful in-service activities.
- Provide support for implementation.
- Visit classrooms and schools in order to understand, first hand, the new teaching methods, programs, and equipment being used.
- Support educational decisions.
- Provide resources and funding for programs.

School Councils

School councils at each school building have potential to make major contributions to school improvement. Appropriate activities could include:

- Study the frameworks and standards.
- Assure that the district provides meaningful in-service activities.
- Provide support for implementation.
- Visit classrooms and schools in order to understand, first hand, the new teaching methods, programs, and equipment being used.
- Support educational decisions.
- Focus the School Improvement Plan on implementation of the Frameworks.

Appendix 3: Chart for Planning and Evaluating Progress in Systemic Change

Determine existing local conditions and potential resources	Establish goals and create an action plan to meet them	Continuously evaluate progress on aspects of the plan
Continue to assess conditions and progress, and revise the plan		

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Massachusetts Curriculum Frameworks in Mathematics and in Science & Technology

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Curriculum Frameworks Response Form

Please take time to review this document and provide us with your comments. Your responses are very important and will help to direct the revisions which will be made by the framework revision committee. Please write your comments on this form. Attach a separate sheet if needed. **Return your comments by January 9, 1995 to:**

PALMS
Attn: Peg Bondorew and Mike Zapantis
Massachusetts Department of Education
350 Main Street • Malden, MA 02148

I. Reviewer Profile

Please supply the following information in the space provided.

Name (optional) or institution or group ________________________________

Address (optional)____

____

Date of Draft ___________________________

Note: If this response form is from a group or organization, please indicate the number represented in each category below. Individuals, please check appropriate categories.

Gender: ____Female ____Male

Race/Ethnicity: ____African American ____White ____Latino ____Native American
____Asian/Pacific Islander ____Other (please specify) ____________

____PreK - Gr. 4 Teacher ____Principal ____Parent

____Gr. 5 - 8 Teacher ____Math/Science Supervisor ____Community Member

____Gr. 9 - 12 Teacher ____District Supervisor ____Business

____ABE Teacher ____University Instructor ____Museum Educator

____Other:_______________________________

II. Framework reviewed: ____Mathematics ____Science and Technology

Introduction: A Vision of Mathematics and Science Education

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about the introduction are:

My suggestions for improvement are:

Chapter 1: Critical Issues of the Framework

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 2: Teaching, Learning, and Assessing Mathematics/Science

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 3: Mathematics or Science and Technology (depending on Framework)

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 4: Implementing the Framework

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Entire Framework

I read about _____ % of the document carefully. My overall reaction is:

Things I like about the draft:

My suggestions for improvement are:

Other Comments:

Request for Vignettes

In order to share rich instructional practices and bring to life the content, pedagogy and partnerships called for in the Curriculum Frameworks in Mathematics and in Science and Technology, the PALMS Curriculum Framework Committee continues to seek vignettes. Many of the vignettes which have been incorporated in this Review Draft were received as a result of earlier requests. In reading this Draft, you will find indications of the need for specific vignettes, and you may also see places where a vignette or example is not called for, but would fit nicely. Perhaps you have a vignette, or an idea for one, which you think belongs in the Frameworks, but you are not sure where it should be placed. We welcome all of your ideas.

Tips on Writing Vignettes

- Describe a classroom experience (or slight modification of a single event or series) to illustrate recommended instructional practices. Tell how students learned important mathematics and science. Share personal insight gained through this experience.

- Include actual dialogue between teacher and student or student and students. Make your story as believable as possible without distorting facts.

- Explain some of the important mathematical ideas that are imbedded in the vignette. The examples may incorporate content from more than one standard or discipline, illustrate how the students encountered the content, how they shared and grappled with misperceptions, and/or how they were able to derive some generalizations. Highlight teacher support and guidance.

- Include what the teacher was doing and thinking during the lesson. Why did the teacher make certain instructional decisions? How did the teacher interact with the students? What kinds of assessment information was gathered?

- As much as possible, describe experiences that include diverse groups of students and address issues of equity.

Vignettes may include one or more of the following elements:

- teacher as facilitator
- cooperative learning
- projects/investigation
- family mathematics/science
- student-to-student interactions
- use of manipulative materials
- learning styles
- use of technology
- community relations
- assessment
- relationship to National and Massachusetts Standards
- interdisciplinary learning

Mail vignettes to: PALMS
c/o Peg Bondorew/Mike Zapantis
Massachusetts Department of Education
350 Main Street • Malden, MA 02148-5023

For further information, please call (617)388-3300 extension 303

Massachusetts

Department of
Education

Massachusetts Curriculum Frameworks for Mathematics

Review Draft

October 1994

NOTE: The attached material is still being discussed by the Massachusetts Department of Education and does not represent Massachusetts Department or Board of Education Policy.

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- Massachusetts Department of Education
- United States Department of Education
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Massachusetts Curriculum Frameworks for Mathematics

Review Draft

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October 1994

Partnerships Advancing the Learning of Mathematics and Science
Massachusetts Department of Education
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Table of Contents

The Commonwealth of Massachusetts
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obert V. Antonucci
Commissioner

October, 1994

Dear Colleagues and Community Members:

The Massachusetts Department of Education recently completed drafts of the Curriculum Frameworks in Mathematics and in Science & Technology. These frameworks are guides that will be especially useful to district curriculum supervisors, and can also be used by teachers, schools, and institutions of higher education to assist in the planning and evaluation of programs. The frameworks implement and support the Common Core of Learning which identifies broad, measurable results for students.

Both documents have been prepared by two writing teams under the direction of PALMS, Partnerships Advancing the Learning of Mathematics and Science, our statewide initiative. The drafts were previewed by a panel of mathematics, science and Department professionals earlier this year. Revisions have been made based on their recommendations.

Although the frameworks are still under review, we are now releasing them for dissemination in your school district. I encourage you to reprint each framework so that copies are available for every building principal, and mathematics and science department heads. We will be notifying all teachers of this process.

Public responses to the Mathematics and Science & Technology Frameworks will be collected through January 9, 1995. We will conduct regional forums as well as focus groups to gather suggestions and build consensus. A survey form is also enclosed within the frameworks for your convenience.

I look forward to hearing your comments and especially look forward to continuing our work together as we strengthen public education in Massachusetts.

Sincerely,

Robert V. Antonucci

Robert V. Antonucci
Commissioner

Acknowledgements

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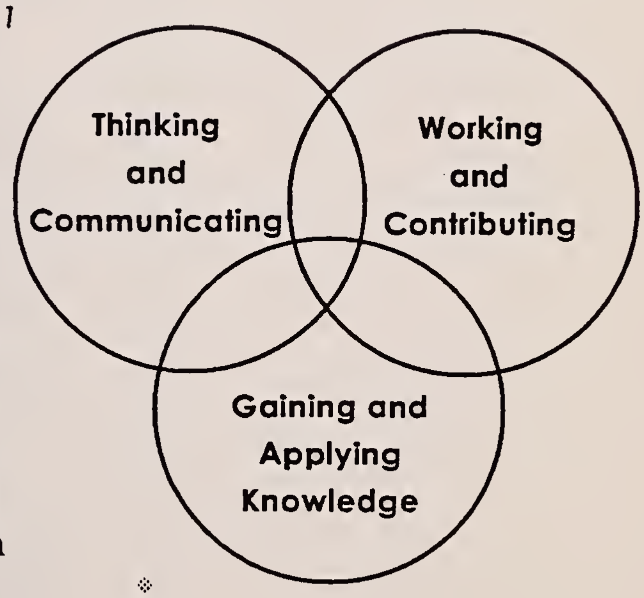
Chapter 3: Mathematics

"Mathematics is the science of patterns and relationships."
(AAAS, Science for all Americans, 1994, p.16)

A major goal of this framework is for all students to develop mathematical power. Developing mathematical power is a complex process. In school, the mathematics children learn depends not only on *what* is taught, but also on *how* it is encountered. In other words, the curriculum cannot be separated from the instructional practices used to teach it. The instructional strategies must encourage students to engage intellectually with important mathematical ideas, embrace the aesthetic value of mathematics, and use mathematics principles to solve problems in their daily lives.

> *Mathematical power includes the ability to explore, conjecture, and reason logically; to solve non-routine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power.*
>
> *NCTM, Professional Standards for Teaching Mathematics, 1991, p. 1*



Mathematical power enables one to continue to gain and apply mathematical knowledge, to think and communicate effectively, to work and be a contributing member of society.

The vision for mathematics classrooms in Massachusetts can only become a reality if students are allowed to construct their own understanding of mathematics from activities designed to promote inquiry. The teacher's role in this process is central. The teacher designs lessons that are rich in mathematics and are often drawn from the students' own experience. An atmosphere is created in which students are free to take risks, to express and defend their ideas. Students are encouraged to work in teams, and to test and verify ideas that emerge from their work among themselves, rather than looking only to the teacher for the final

validation. In this setting students learn from each others' explanations and from the process of clarifying their own understanding as their peers question their findings.

Lessons that a teacher introduces to a class should do at least three things: target important mathematical ideas; promote the connections among mathematical ideas; and identify relationships between the ideas introduced in the lesson and the concepts with which students are already familiar. The mathematical content of each lesson and activity needs to be identified and defined by the teacher before introducing the content to the students. It is essential that each lesson end with careful discussion of the concepts that the students explored and of the relationships uncovered in the process. This discussion is important for several reasons. One is that it permits students' to clarify their learning and integrate it more firmly into their existing network of ideas. Another is that it serves as a tool for ongoing evaluation by the teacher. The teacher needs to carefully compare the network of ideas included in the curriculum with the students' knowledge as they discuss what they have learned. The teacher needs to note discrepancies, misunderstandings, and gaps in students' knowledge as well as evidence of learning. Finally, new lessons and experiences must be designed to test students' false assumptions and conclusions, confirm accurate findings, and extend the students' knowledge. Educators must assure that substantial gaps do not develop in the system of mathematical knowledge which students are developing, but that a rich matrix of ideas has been thoroughly explored at the end of each academic year.

These steps for building lessons are critical for the success of any teacher involved in assisting students to explore and discover mathematics. The emphasis throughout this document is on the essential processes of mathematical inquiry: problem solving, communication, reasoning and establishing connections. Simultaneously, teachers must remember the importance of delivering to students, through these processes, a substantial understanding of the content of mathematics.

This chapter presents the mathematics all students should know and be able to do. The description of content is integrated with how students are to work with and use the mathematics. All the vignettes and activities incorporate teaching strategies with activities that engage student interest while working on important mathematical content.

Toward an Integrated Mathematics Curriculum

As noted earlier in Chapter 2, the Framework recommends an interdisciplinary approach to learning. Understanding the world, requires, for example, relating students' knowledge in algebra to what students know of science, relating art and geometry, and social studies and statistics. A more integrated approach within the mathematics curriculum is also recommended. This approach is reinforced throughout this chapter.

In the elementary grades, an integrated approach to mathematics might include activities which combine measurement, estimation, and geometry. In middle schools and high schools it will mean helping students make connections between ideas from algebra and geometry, but also among ideas from discrete mathematics, probability, and statistics. This recommendation for a more integrated mathematics curriculum is made because:

- It will provide students with a more accurate picture of the nature of mathematics and will give them the opportunity to understand the essential connections among various fields of mathematics.
- Students will become more powerful problem solvers if they learn that problems can be approached in more than one way. Problems in the world do not come labeled "geometry" or "trigonometry."
- If students are continually encountering ideas from all parts of mathematics, it is likely that less review time will be required, hence there will be more time to teach some of the recommended mathematics topics, such as statistics and probability, which have not been part of the traditional mathematics curriculum for all students.
- Presenting problems which can be solved in more than one way will allow students to rely on their strengths and, simultaneously, develop new abilities. A problem which can be approached either visually or numerically, for example, will be accessible to those students in the class who are visual learners. At the same time, as solutions are shared, they will have an opportunity to learn how the problem can be approached numerically. The same kind of sharing is possible for students who are more comfortable with numerical approaches and need to develop their visual skills.

Part of the task for schools and school systems will be to develop an approach to a more integrated curriculum which is appropriate for the local circumstances. At the secondary level, where courses traditionally have been organized by content area, this will require thoughtful consideration of connections between major ideas.

Some schools may seize every opportunity within existing courses to show connections within mathematics. Systems which adopt this approach will need to provide time for teachers to discuss with one another the content of their courses, the connections they are making, and the responses of their students. Other systems may select textbooks which present an integrated approach. In this case it will be important to conduct a careful review of textbooks. Some series merely include short sections on isolated topics, alternating between chapters on algebra and geometry, for example. In either case, schools will find that technology can make important contributions, making it possible, for example, to switch from equations to graphs to data analysis.

As has been stated with respect to many other issues in this framework, ongoing relevant professional development is an essential key to progress in promoting an integrated mathematics curriculum. In this as in many other areas, business partners can be of particular help, both in providing examples of problems which require the use of a variety of mathematical topics and tools, and in providing more direct assistance to teachers and students.

Equity Framework in Mathematics, Science and Technology Education

The under-representation of certain groups (including African-American, Latino, Native American, female, and economically disadvantaged students; as well as students with learning disabilities and those who are physically challenged) in mathematics, science and technology education is well documented. The National Science Foundation Statewide Systemic Initiative program is designed to improve mathematics, science and technology education for all students. One of the criteria for award to the SSI is the inclusion of a statewide strategy for addressing issues of equity in their program design. One of the SSI goals is to effect change in student performance by changing school curriculum, classroom instruction, teacher education, and parent involvement to ensure that all students are provided equitable opportunities to learn mathematics, science and technology. A major focus is to serve under-represented students, eliminating performance gaps while raising the level of knowledge and skills in mathematics and science for all students.

Equity Framework developed by the Forum for the Statewide Systemic Initiatives Draft 08-09-94

The issue of equity has been discussed in other parts of the frameworks but because of its critical importance to the realization of our vision for mathematics education for the students of Massachusetts, it should be reemphasized. It is not enough for us to say that all children can learn mathematics. Educators and the community must do everything necessary to insure the success of every student. Different expectations of students can lead to reduced performance and restricted development that can have a direct influence on future career choices. It is not enough to just enroll students in higher level classes; we must do everything possible to engage their interest. They should not be allowed to sit on the periphery of the class and not participate. Simply mixing children together will not solve the problem of tracking.

Gender Vignette

Recent press surrounding gender issues brought the topic to the floor of one of my eighth grade classrooms. Students were surprised that such prejudice does exist. (My relief was noticeable.) It was, however, a great time to "seize the moment" and suggest that maybe we should conduct some kind of investigation to see if their perceptions of my lack of bias were accurate. We decided that a sample of just that classroom would not suffice. At least the whole team of 92 would have to be involved. For two weeks students monitored the number of girls versus the number of boys on whom I called. Because there were so many more boys than girls in one class, they were quick to decide that ratios were needed (daily, because of absentees). When it seemed that I, indeed, may have been favoring the boys at one point, the girls surprisingly enough wanted me to feel that it was "OK" because, after all, the boys did have their hands up more often. When I responded that I would still want to be sure that the girls had equal opportunity, they continued the score keeping and ratio tallying with added zeal. The issue really did give new meaning to the need for ratios, as well as a new dimension of collegiality to the spirit of the classroom. -- *Marcia Harol*

Grouping for Mathematics Instruction

One of the most complex and difficult questions for schools and school systems is, "How should students be grouped in order to promote the learning of mathematics for all students?" Students learn mathematics at different rates, and different students' interests in mathematics vary. We want all students to achieve to their best potential in mathematics, with none being "turned off" because the

progression of the class is too slow, or because the students lack prerequisite mathematical understandings. In the past, a common way of addressing the needs of different students has been through homogeneous grouping—grouping students together on the basis of achievement and/or interest.

There are several ways students' differing needs have been accommodated including tracking, homogeneous grouping, and heterogeneous grouping. The following definitions are provided for discussion purposes.

- **Tracking** is the placement of students into groups according to early evaluations of the students' abilities. A student is in the same group no matter what the subject. The student is unable to move out of these groups despite any growth or problems he or she may experience.
- **Homogeneous grouping** is the placement of students into groups according to the abilities of the students in each subject. As a student's progress or needs dictate, the student could change groups for different subjects.
- **Heterogeneous** grouping is the placement of students into groups without regard to their abilities. All ability levels are present in a classroom.

As stated in Chapter 1, the developers of this Framework concur with the position that no tracking should occur in schools. There are, however, situations where both homogeneous and heterogeneous groupings may be appropriate.

In elementary schools, students are generally placed in heterogeneous classrooms; the primary criterion for placement is the student's age or grade level. Within a classroom, a teacher may group students in mathematics for a number of reasons: students may be working together on different projects, the teacher may work with a group of students who need special help, or highly motivated students may be working on enrichment activities. Some groupings may be temporary and some may be of longer duration.

In middle schools, for the most part, students should be placed in heterogeneous mathematics classes. As in elementary school, there are times when students benefit from working in homogeneous groups, as long as that grouping does not become automatic and institutionalized. Opportunities for extra help should be available for all students based on individual needs. Appropriate opportunities for enrichment and/or advancement need to be provided for students who are highly talented and/or motivated in mathematics.

At the high school level, both homogeneous and heterogeneous grouping of students can work. Students should be grouped into mathematics courses based on their interest and ability to do mathematics. Not all students are ready to learn the same mathematics content at the same time. There can be more than one pathway for students to develop mathematical power. For example, students who are preparing to study technological areas require different mathematical content than students who are preparing to study the humanities.

While students at the secondary level may be enrolled in different courses, the mathematics should be challenging and expectations for all students should be high. The content of all mathematics courses should be organized around a core curriculum as identified in the NCTM *Curriculum and Evaluation Standards* and this Framework. The curriculum should be "differentiated by the depth and breadth of the treatment of topics and by the nature of applications." For all students, "the 9–12 *Standards* call for a shift of emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understandings, multiple representations and connection, mathematical modeling, and mathematical problem solving" (NCTM, 1990, p. 125).

There are many risks in providing different courses for different groups of secondary students. One risk is that students in different courses will not be treated equitably. Students in all courses need equal access to tools for learning mathematics. These tools include, among others, measuring instruments, manipulatives, graphing calculators and computers. As recommended in Chapter 1 of this Framework, all students should to take a high quality mathematics course each year.

Standards for Mathematics

The *Curriculum and Evaluation Standards for School Mathematics*, developed by the National Council of Teachers of Mathematics (NCTM), represent the consensus of the nation's mathematics education community about the fundamental content that should be included in the school mathematics curriculum. The NCTM Standards are the basis for the Massachusetts Mathematics Curriculum Framework. The *Standards* should be used by districts, schools and teachers as they examine their current mathematics program and as they develop new curriculum specifications.

The importance of mathematical literacy must be addressed as new curriculum is developed. The NCTM *Standards* (pp.5-6) specify five educational goals for all students. They are:

- Students learn to value mathematics
- Students become confident in their ability to do mathematics
- Students become mathematical problem solvers
- Students learn to communicate mathematically
- Students learn to reason mathematically

In order to assist readers in thinking about and designing their curriculum, this framework organizes the NCTM *Standards* into two sets or clusters. The following four standards are called the "Fundamental Standards:

- Mathematics as Problem Solving
- Mathematics as Communication
- Mathematics as Reasoning
- Mathematical Connections

These standards should be embedded in all the mathematics work students do. They permeate all topics and all three grade level spans: pre-K-4, 5-8, and 9-12.

The remaining NCTM *Standards* are organized into the following clusters of major ideas:

I Number Sense

A. Numbers and Number Systems

B. Estimation and Computation

II Patterns, Relations and Functions

III Geometry and Measurement

IV Probability and Statistics

The clustering was done in order to emphasize the connectedness among the NCTM Standards and to assist educators who are working on curriculum revision. While any grouping is somewhat arbitrary, it is hoped that the readers will find this organization helpful. No standard uniquely belongs to one and only one cluster; many standards cut across several clusters. For example, even though Discrete Mathematics is listed under cluster IA, there are concepts from discrete mathematics that are embedded in every cluster; and while this standard is identified as for grades 9-12, there are also discrete mathematics concepts appropriate for every grade level, K-12 and beyond. The same is true for the other standards.

The table below shows how the *NCTM Standards* are clustered in this Framework. Clustering the standards is intended to show some of the connections among the standards and the continuity of the standards across the grade levels. Using the clusters, the Framework tries to show connections between topics and across levels. Although the standards in this Framework are organized by topic and grade span, in actual practice these standards will not be taught in isolation, but rather in an integrated fashion.

The Massachusetts Adult Basic Education Math Standards, (1994) which extend the *NCTM Standards* to the field of adult education, have also been included in this chart.

The Framework uses this organizational scheme to discuss the important mathematics all students should know and be able to do. Following a discussion of the Fundamental Standards, each cluster is introduced with a rationale that cuts across grade levels, pre-K-12 and adult basic education. While it is important for teachers to understand what should be happening at their level, it is also critical that districts, schools, and teachers see the how mathematics content is to flow across a broad range of topics and grades.

Following the K-12 rationales for each cluster, a number of classroom activities and vignettes are included by grade level span, pre-K-4, 5-8, 9-12, and ABE. These illustrate how students might encounter and engage with some of the mathematical ideas in school. The activities provide a range of ideas for teachers to use with their students. The vignettes are intended to provide brief glimpses students and teachers interacting and doing mathematics together in classrooms.

Fundamental Standards

The first four *NCTM Standards*, often referred to as the "process" standards, are common across all grade levels. They are:

1. Mathematics as Problem Solving

2. Mathematics as Communication

3. Mathematics as Reasoning

4. Mathematical Connections

These standards are cut across all other standards. They also overlap and interrelate with one another. Incorporating more problem solving in the mathematics curriculum, for example, will provide a reason for students to work together and communicate with one another. Encouraging students to communicate will provide opportunities for them to explain their reasoning, and to listen to and understand the reasoning of others. As students explain their reasoning and methods of solution to one another, they may see, for example, that some students' solutions are based on diagrams while others' are based on number patterns—that there are connections between geometry and algebra. All work students do in mathematics should reflect these standards. They represent the essential processes that should be characteristic of all mathematical teaching and learning. These standards are embedded in all classroom activities and vignettes that are included in this chapter.

These standards represent a shift from traditional ways of thinking about mathematics education. The table below contrasts the traditional and new paradigms as they relate to the four Fundamental Standards.

NCTM Curriculum Standards

Fundamental Standards
1. Mathematics as Problem Solving 2. Mathematics as Communication 3. Mathematics as Reasoning 4. Mathematics as Connections

Cluster	K-4 NCTM Standards	5-8 NCTM Standards	9-12 NCTM Standards
I Number Sense A. Numbers and Number Systems	(6) Number Sense and Numeration (7) Concepts of Whole Number Operations (12) Fractions and Decimals	(5) Number and Number Relationships (6) Number Systems and Number Theory	(12) Discrete Mathematics (14) Mathematical Structure
B. Computation and Estimation	(5) Estimation (8) Whole Number Computation	(7) Estimation and Computation	
	ABE Standards	(5) Estimation (6) Numbers, Operations, and Computation	
II Patterns Relations and Functions	(13) Patterns and Relationships	(8) Patterns and Functions (9) Algebra	(5) Algebra (6) Functions (9) Trigonometry (13) Conceptual Underpinnings of Calculus
	ABE Standards	(7) Patterns, Relationships, and Functions (8) Algebra	
III Geometry and Measurement	(9) Geometry and Spatial Sense (10) Measurement	(12) Geometry (13) Measurement	(7) Geometry from a Synthetic Perspective (8) Geometry from an Algebraic Perspective
	ABE Standards	(10) Geometry and Spatial Sense (11) Measurement	
IV Statistics and Probability	(11) Statistics and Probability	(10) Statistics (11) Probability	(10) Statistics (11) Probability
	ABE Standards	(9) Statistics and Probability	

Note: The numbers in parentheses indicate the standard number used in the NCTM Curriculum and Evaluation Standards for School Mathematics.

	Traditional Paradigm	New Paradigm
Problem Solving	Problem solving means solving word problems. Problems are, in fact, computational exercises embedded in a simple context. They are done following introduction and practice of a specific procedure to reinforce the procedure and to provide examples of where the procedure is used. There is an assumption that complex problems are OK for "advanced" students, but too difficult for "slower" students, only few students are expected to attempt the more complex problems.	Problem-solving is the focus of mathematics programs. All students are capable of solving problems, but need many opportunities to solve problems. Students need to develop and use a variety of strategies in solving problems, and understand the use of multiple strategies to solve the same problem. They must feel that there are multiple solutions to problems, or multiple pathways to solutions.
Communication	The primary means of communicating mathematics is one-way: from the teacher (or text book) to the students. Students are expected to communicate their knowledge of mathematics by using taught procedures in response to questions on tests. Students are expected to memorize mathematical definitions.	Students learn when they reflect on ideas and communicate their thoughts with others. In mathematics classes, students frequently interact with other students to solve problems and to sharing their strategies and seeking solutions to problems. Students frequently explain and justify their thinking in writing. They come to understand mathematical terms by comparing and contrasting examples with other students before a formal definition is presented.
Reasoning	While pure mathematics involves reasoning, learning basic mathematics in school requires memorization of rules and procedures. Understanding can be developed through teacher explanation, and, for elementary students, the use of manipulative models. The basics need to be "mastered" first, so the students have some content knowledge to reason with. Most students aren't mature enough to do complex or abstract reasoning.	All students are expected to reason about mathematics, respond to the reasoning of other students and the teacher and communicate their reasoning to others. Routinely, students make conjectures, think about and select sensible ways to solve problems, and justify their solutions. They should recognize different types of reasoning as they use them, and understand the types of certainty associated with deduction, induction, analogy and statistical reasoning.
Connections	Mathematics content is broken down into several large strands, such as arithmetic and geometry, and the content of each strand is organized into a series of small, sequential objectives. Each of these strands is taught separately, without an effort to explore the commonalties in reasoning and uses of the skills in different strands. The learning of new content most often involves memorization of new facts and formulas. As a result, students learn that mathematics is a collection of many isolated topics, rules, and procedures.	While mathematics content is organized into separate *standards*, students need to make connections among different concepts within mathematics. Connections should also be made between mathematical ideas and other subject matter disciplines. Perhaps the most important connection to be fostered in mathematics instruction is the connection between the mathematical ideas and students' experiences within a real world context.

Standard 1: Mathematics as Problem Solving

Problem solving may be defined as applying knowledge, skills, and experiences in efforts to resolve a dilemma or situation that is new or perplexing. "Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic, but a process that should permeate the entire program and provide the context in which concepts and skills can be learned." (NCTM 1989, p. 23)

Problem solving as discussed here means something quite different from typical word problems. The following two problems illustrate that most problems that are encountered in real life are quite complex—there are many factors affecting the situation, there are many ways to view and approach the problem, there are many strategies for solving the problem, there is frequently more than one solution, and several criteria for what is the "best" solution.

- How do I choose the best car for my family? Should I buy now or wait another year and risk substantial repair bills? What are the most important factors for me to consider in selecting a car? What weight should I give to each factor?

- I have a fairly secure, steady job with a stable company, but there seems to be little chance for advancement. Should I invest in myself and return to school? Full-time or part-time? Should I apply for a position in a fast-growing but unproven company? How do I calculate the estimated costs and probable benefits?

To become good problem solvers, students need to have many opportunities to create and solve problems in both mathematical and real world contexts. Students need to pose questions, define problems, consider different strategies, and find appropriate solutions. By sharing various approaches, strategies and solutions, students realize that there are many ways of looking at and solving a problem, and that some ways are more effective and/or efficient than others in particular circumstances. In addition, they develop self confidence as competent problem solvers.

Standard 2: Mathematics as Communication

Mathematics can be thought of as a language "because of its power to represent and communicate ideas concisely." (*NCTM*, 1989, p. 78) It is a language that is important for students to learn so they are able to make sense of mathematical situations and to communicate about these situations with others. Learning how to look at a real situation from a mathematical perspective, talk about the mathematics of the situation, translate the everyday language into mathematical symbols and notation in order to find a mathematical solution, and then interpret the solution in the context of the original situation is an important part of learning mathematics. Learning the language of mathematics develops gradually, and has to be continually connected to what students understand to make it meaningful.

Communication also is important in mathematics since it is a tool for learning. Students learn mathematics as they talk and write about what they are doing. Students become actively engaged in doing mathematics when they are asked to think about their ideas, talk with and listen to other students, sharing different ideas, strategies, and solutions. Writing about mathematics helps students reflect on their work and clarify ideas for themselves. Writing also is a way for teachers to identify students' understandings and misconceptions.

Students will also benefit from opportunities to learn to read mathematics—to read something someone else has written about content they already understand, and later to read in order to learn new content.

Standard 3: Mathematics as Reasoning

"Reasoning is fundamental to the knowing and doing of mathematics." (*NCTM*, 1989, p. 81). The ability to reason enables students to solve problems, and it also contributes to their senses of confidence that they can do mathematics by validating their own thinking.

Mathematics as a field of study is characterized as much by particular types of reasoning as by particular types of content. Students need to learn to recognize and use deduction and induction, and develop the ability to apply these in numerical and spatial contexts. They should learn to evaluate arguments and establish their validity, to analyze what makes sense, and what is true, or plausible but not true.

Students should be encouraged to reflect on and to articulate their reasoning by questions such as, "How did you figure that out?" "Why is this a good way to solve the problem?" "Are there other ways?" "Can you think of them?" "Can you be sure you have the right answer?" "Can there be more than one right answer?" "How would you explain your solution if you wanted to convince someone else that it was correct?" Students should be encouraged to make conjectures, test them, and determine whether they can be shown to be true. Asking students to share their thinking about how they worked on a problem often is a way to help students identify their own mistakes or flaws in reasoning.

Standard 4: Mathematical Connections

It is important that students see the relations and connections among mathematical ideas—that they view mathematics as an integrated whole. Students need to experience and understand that in using mathematics to solve or explore problems and topics which evolve from real situations, and which evolve from other mathematical ideas, they may draw on any or several areas of mathematics. Novel, important, or interesting problems do not come with labels such as "geometry," "algebra," or "probability." Part of the process of problem solving is to figure out what ideas from geometry or algebra or probability, for example, might be useful.

Students need to recognize equivalent representations of mathematical concepts, equivalent procedures for performing an operation or solving a problem, and the extensions of ideas in one area of mathematics into other areas. For example, the Pythagorean theorem can be viewed from different mathematical perspectives—it can be represented using concrete manipulatives, algebra, geometry, and computer simulations.

While mathematical content as presented in the NCTM *Standards* and this Framework is divided into distinct standards, topics, and clusters, these descriptions serve only as an aid in identifying appropriate content for students in K-12. The content need not be presented to students in the way the descriptions are organized.

Finally it is important for students to understand that mathematics is related to other subjects they study—art, music, social studies, health, and physical education, for example. They need to learn that mathematics is one way of learning about the world, and it is connected, not isolated, from other ways of learning. In addition, these connections will help students use their strengths and interests to branch out to subjects with which they feel less comfortable. A proficient mathematics student who thinks of him or herself as someone who "can't draw" may be engaged in and helped to explore art through the study of tessellations in mathematics class. A history buff who thinks he or she can't do mathematics may gain a new view of mathematics and his or her ability in mathematics by analyzing data from a questionnaire on family histories.

NCTM Standards 1–2

NCTM Standard 1: Mathematics as Problem Solving

In grades K-4 the study of mathematics should emphasize problem solving so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningful.

In grades 5-8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from situations within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multistep and non routine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations;
- acquire confidence in using mathematics meaningfully.

In grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can–

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multistep and non routine problems;
- recognize and formulate problems from situation within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

ABE Standard 1: Mathematics as Problem Solving

In the adult basic education classroom, curriculum design must include approaches which allow the learner to:

- explore and employ multiple strategies for solving problems;
- determine, collect, and analyze appropriate data with respect to the original problem or in new problem solving situations;
- have access to and the ability to use appropriate problem solving tools including the use of calculator, computers, and measurement instruments;
- generalize problem solving strategies to a wide range of adult-oriented, real-world situations.

NCTM Standard 2: Mathematics as Communication

In grades K-4 the study of mathematics should include numerous opportunities for communication so that students can—

- relate physical materials, pictures, and diagrams to mathematical ideas;
- reflect on and clarify their thinking about mathematical ideas and situations;
- relate their everyday language to mathematical language and symbols;
- realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.

In grades 5-8 the study of mathematics should include opportunities communicate so that students can—

- model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
- reflect on and clarify their own thinking about mathematical ideas and situations;
- develop common understandings of mathematical ideas, including the role of definitions;
- use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
- discuss mathematical ideas and make conjectures and convincing arguments;
- appreciate the value of mathematical notation and its role in the development of mathematical ideas.

In grades 9-12 the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can—

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

ABE Standard 2: Mathematics as Communication

In the adult basic education classroom, curriculum design just include approaches to teaching mathematics as communications which allow the learner to:

- develop appropriate reading, writing, listening and speaking skills necessary for communicating mathematically in a variety of settings;
- discuss with others, reflect and clarify their own thinking about mathematical outcomes, and make convincing arguments and decisions based on these experiences.
- define everyday, work-related or test-related mathematical situations using concrete, pictorial, graphical or algebraic methods;
- appreciate the value of mathematical language and notation in relation to mathematical ideas.

NCTM Standards 3–4

NCTM Standard 3: Mathematics as Reasoning

In grades K-4 the study of mathematics should emphasize reasoning so that students can—
- draw logical conclusions about mathematics;
- use models, known facts, properties, and relationships to explain their thinking;
- justify their answers and solution processes;
- use patterns and relationships to analyze mathematical situations;
- believe that mathematics makes sense.

In grades 5-8, reasoning shall permeate the mathematics curriculum so that students can—
- recognize and apply deductive and inductive reasoning;
- understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
- make and evaluate mathematical conjectures and arguments;
- validate their own thinking;
- appreciate the pervasive use and power of reasoning as a part of mathematics.

In grades 9-10, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can—
- make and test conjectures;
- formulate counter examples;
- follow logical arguments;

In grades 11-12, the mathematics curriculum should reinforce and extend logical reasoning skills so that all students can–
- judge the validity of arguments;
- construct simple valid arguments;

and so that, in addition, college-intending students can—
- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

ABE Standard 3: Mathematics as Reasoning

In the adult basic education classroom, curriculum design must include approaches which emphasize mathematical reasoning so that the learner can:
- draw logical conclusions from mathematical situations using concrete models and verbal skills to explain their thinking;
- understand and apply deductive and inductive reasoning, proportional reasoning, with special attention to spatial and visual reasoning with proportions and graphs;
- pose their own mathematical questions and evaluate their own arguments;
- validate their own thinking and intuition, feel confident as math problem solvers, and see that mathematics makes sense.

NCTM Standard 4: Mathematics as Connections

In grades K-4 the study of mathematics should include opportunities to make connections so that students can—
- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas;
- use mathematics in their daily lives.

In grades 5-8, the mathematics curriculum should include the investigation of mathematical connections so that students can—
- see mathematics as an integrated whole;
- explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations;
- use a mathematical idea to further their understanding of other mathematical ideas;
- apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business;
- value the role of mathematics in our culture and society.

In grades 9-12, the mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications that all students can—
- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;
- use and value the connections among mathematical topics;
- use and value the connections between mathematics and other disciplines.

ABE Standard 4: Mathematics as Connections

In the adult basic education classroom, curriculum design must include approaches to making mathematical connections which allow the learner to:
- view mathematics as an integrated whole that is connected to past learning, the real world, adult life skills, and work-related settings;
- explore problems using appropriate technology and describe results using a variety of mathematical models or representations including graphs, concrete, verbal, and algebraic models or representation;
- apply mathematical thinking and modeling to solve problems that arise in other disciplines, and in the real world, including work-related settings.

Cluster I: Number Sense

Number sense is the cornerstone in using mathematics to understand the world around us; it is the development of an understanding of the various uses of numbers and the relationships between them. A sound understanding of number empowers people to interpret and represent the world in which they live. For a child this may represent the ability to share a pile of strawberries fairly with a friend; for a more advanced student it may be using ratios to understand the inequity represented by the low percentage of African-Americans and Latinos graduating from college with degrees in mathematics and science.

Number is the basis for much of what students study in mathematics. The need for numbers, to count and to measure, is as old as civilization. Nonetheless, many people have poorly developed number sense—the ability to sensibly interpret and use numbers in order to understand issues and accomplish tasks they need to do. The development of number sense begins before formal education and continues through adulthood. For example, adults struggle with comprehending the significance of concepts such as population growth, light years, or a budget deficit in trillions of dollars. A lack of number sense can contribute to people feeling incapable of participating in discussion of complex social issues, for example, what would a new health care system cost?

Lauren Resnick describes number sense in the following way.

> How can number sense be defined? . . . It turns out to be nearly impossible, because number sense, upon reflection, is not a collection of things that one knows about numbers or of skills that one can exercise upon numbers. Rather, it is a set of not fully predictable things that one tends to do with numbers under certain circumstances on the basis of a body of interrelated concepts of number and knowledge of specific numbers.
>
> The following is a list of possible indicators (not components) of number sense:
>
> - *Using well-known number facts to figure out facts of which one is not so sure. Note that this will be observed only when the particular individuals involved feel they know the "benchmark" number fact much better than the fact to be figured out, when they value accuracy more highly than speed, and when there is no more efficient procedure (such as counting) available to them.*

- *Judging whether a particular number constitutes a reasonable answer to a particular problem. Note that someone might be able to judge what is reasonable in some situations, but not in others. Note also that we would normally see this kind of number sense only when a person has generated a wrong answer.*

- *Approximating numerical answers (rather than calculating exact answers). Note that this will be used only when the individual judges that an approximation is adequate and when it is easier (quicker, more reliable) for that person to approximate than to calculate exactly. The latter may depend, in part, on whether pencil and paper or a calculator is available.*

- *Tending to want to "make sense" of situations involving number and quantity. Talking about numbers and their relationships.*

- *Having a sense of the relative size of numbers and the quantities to which numbers refer. Note that the values of numbers depend on the situational context; the same number can refer to "a lot" or "a little" in different situations.*

- *Substituting flexibly among different possible representations of a quantity (e.g., 24 for 2 dozen, a little less than 1/2 for 0.4). Note that a substitution's usefulness depends on the particular problem to be solved and on details of the individual's knowledge of numbers.*

Resnick, 1989, p. 36

An important role of number sense is in the development of mathematical intuition, which is based on familiarity and extensive experience with numbers and their relationships. Mathematical intuition often plays a significant role in mathematical problem solving. A strategy for solving a problem may leap to mind instantaneously, or may suddenly emerge after taking a fresh look it, before the student has considered or applied more formal mathematical methods. The benefits of mathematical intuition and facility with numbers also extends into areas of advanced mathematics.

Number sense is complex and is central to several of the NCTM *Standards*. To assist the reader, this cluster of standards has been divided into two parts: "A. Number and Number Systems," and "B. Estimation and Computation."

A. Number and Number Systems

The need for an understanding of number begins with a child's desire to quantify the world by counting and comparing counts of different things. A very young child, when given two cookies will often ask for "more" although the child may not know how to count. A preschool child may be able to recite the counting sequence, yet may have little idea of the notion of "fiveness." To a first grader, one hundred is "a lot." As children mature and gain confidence in using numbers in familiar situations they develop a personal understanding of quantity and quantitative relationships.

An example of how this occurs can be traced through students' development of the concept of number. While children's first use of numbers is with whole numbers, they also naturally encounter, and make sense of, positive rational numbers at an early age. They use fractions when sharing something (half an apple) and decimals when they start to spend money. During the middle grades, students explore negative integers, perhaps through subzero temperature reading in Celsius, to construct a beginning understanding of the system of integers. Students at these grade levels and at high school, expand their understanding of rational numbers when, for example, they use ratios and proportions to build a scale model of their school or City Hall. They also begin to learn about irrational numbers when they investigate the Pythagorean theorem on a geoboard or with dot paper.

Students' use of numbers in different situations over a long period of time is one of the building blocks for their understanding of number systems and the structure of mathematics. As students develop fluency using numbers and computation, they need to develop understandings of number systems and number theory. This can begin very informally, for example, asking a second grader to investigate and explain what happens when odd and even numbers are added together. Students also will investigate primes, composites, factorization, divisors, multiples, and the properties of zero and one. Older students will extend their understanding of operations with whole to integers and with rational to irrational numbers. In high school students will examine in more depth the characteristics of the subsystems of the real number system. Students will come to see that at even higher level of mathematics, counting and measuring concepts are critical—counting is a basic component of discrete mathematics, while measurement is the gateway to ultimately understanding calculus. Understanding the structure of mathematical

systems helps students analyze and make connections among different systems.

It will also be important for students to have experience with mathematical systems that are not continuous, and to understand that there are different mathematical systems that are appropriate for exploring and solving different types of problems. Discrete mathematics is "the study of mathematical properties of sets and systems that have only a finite number of elements" (*NCTM Standards*, p. 178). It includes logical thinking, counting principles, probability, matrices, graph theory, iteration, and recursion. Because many of the topics in discrete mathematics are accessible to students who do not have strong computational skills, discrete mathematics can provide a new entry point into mathematics for these students. Investigating topics from discrete mathematics provides students with a fuller and more accurate view of mathematics, and with a wider range of mathematical tools for defining and solving complex problems.

Adult basic education mathematics students can develop a sense of the magnitude and relative position of numbers through the use of physical materials and "real life" referents. By employing such "realia," connections between fractions, decimals, and percents can be explored, analyzed, and understood. Students in the end should be able to understand the numbers they use in their everyday lives and in the world around them.

For the ESL student, it is important to develop English language and vocabulary skills necessary to express numbers and number concepts. In particular, for students trained in the European method of notation, it means learning the difference between the American use of comma and decimal point and the European use, ie. 0.5 instead of 0,5, and 5,000 instead of 5.000.

Workplace learners must be taught how to determine appropriate vehicles for describing numerical phenomena. They should be given experience with plotting and interpreting numerical relationships in one and two dimensional graphs.

NCTM Number and Number System Standards

K-4 Standards

Standard 6: Number Sense and Numeration
In grades K-4, the mathematics curriculum should include whole number concepts and skills so that students can—

- construct number meanings through real-world experiences and the use of physical materials;
- understand our numeration system by relating counting, grouping, and place-value concepts
- develop number sense;
- interpret the multiple uses of numbers encountered in the real world .

Standard 7: Concepts of Whole Number Operations
In grades K-4 the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can—

- develop meaning for the operations by modeling and discussing a rich variety of problem situations;
- relate the mathematical language and symbolism of operations to problems situations and informal language;
- recognize that a wide variety of problem structures can be represented by a single operation;
- develop operation sense.

Standard 12: Fractions and Decimals
In grades K-4 the mathematics curriculum should include fractions and decimals so that students can—

- develop concepts of fractions, mixed numbers, and decimals;
- develop number sense for fractions and decimals;
- use models to relate fractions to decimals and to find equivalent fractions;
- use models to explore operations on fractions and decimals;
- apply fractions and decimal to problem situations.

5-8 Standards

Standard 5: Number and Number Relationships
In grades 5-8, the mathematics curriculum should include the continued development of number and number relationships so that students can—

- understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problems situations;
- develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;
- understand and apply ratios, proportions, and percents in a wide variety of situations;
- investigate relationships among fractions, decimals, and percents;
- represent numerical relationships in one- and two-dimensional graphs.

Standard 6: Number Systems and Number Theory
In grades 5-8, the mathematics curriculum should include the study of number systems and number theory so that students can—

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

NCTM Number and Number System Standards

9-10 Standards

Standard 12: Discrete Mathematics
In grades 9-10, the mathematics curriculum should include topics from discrete mathematics so that all students can—

- represent problem situations using discrete structures such as finite graphs and sequences;
- develop and analyze algorithms.

Standard 14: Mathematical Structure
In grades 9-10, the mathematics curriculum should include the study of mathematical structure so that all students can—

- compare and contrast the real number system and its various subsystems with regard to their structural characteristics.
- understand the logic of algebraic procedures;

11-12 Standards

Standard 12: Discrete Mathematics
In grades 11-12, the mathematics curriculum should include topics from discrete mathematics so that all students can—

- represent problem situations using discrete structures such as matrices and recurrence relations;
- represent and analyze finite graphs using matrices;
- solve enumeration and finite probability problems;
- investigate problem situations that arise in connection with computer validation and the application of algorithms;

and so that, in addition, college-intending students can—

- represent and solve problems using linear programming and difference equations.

Standard 14: Mathematical Structure
In grades 11-12, the mathematics curriculum should include the study of mathematical structure so that all students can—

- appreciate that seemingly different mathematical systems may be essentially the same;

and so that, in addition, college-intending students can—

- develop the complex number system and demonstrate facility with its operations;
- prove elementary theorems within various mathematical structures, such as groups and fields;
- develop an understanding of the nature and purpose of axiomatic systems.

ABE Standard 6: Numbers, Operations, and Computation
In the adult basic education classroom, curriculum design must include concrete and developmental approaches to teaching numbers, number relationships, operations, and computation which allow the learner to:

- understand, represent, and use numbers in a variety of equivalent forms and in order relations (integers, fraction, decimal, percent, exponential, and scientific notation) in real-world, work-related, and mathematical problem situations;
- compute with whole numbers, fractions, decimals, and integers, using appropriate algorithms and a variety of technique including mental math, paper-and-pencil, calculator, and computer methods;
- analyze and explain procedures for computation and understand how arithmetic operations are related to one another, particularly the reversibility of operations;
- Use estimation to develop number sense, operation sense, and to check the reasonableness of results.
- Analyze and explain methods for solving proportions;
- select and use in problem solving situations an appropriate method from among mental arithmetic, paper-and-pencil, calculator, and computer methods;
- use computation, estimation, and proportions to solve problems.

Grades K-4: Number and Number Systems Vignettes and Activities

Vignette: Sorting Leaves in Kindergarten

On a day in October, Ms. Chapman's class took a walk around the school ground to observe autumn changes. Each student picked out a leaf that had fallen to the ground and brought it back to class. When the students were seated on the rug in a circle with the leaves in the center, Ms. Chapman asked, "What do you notice about the leaves?

Ms. Chapman uses "Autumn Changes" as the basis for interdisciplinary learning experiences in her kindergarten classroom. The students will talk about the seasons, and observe how the change from summer to winter affect plants and animals. They will listen to stories about autumn, and they will illustrate the seasons. This vignette shows how they sort the leaves and do some informal measuring of them.

Soleap said, "There are lots of them."

"How many do you think there are?" asked Ms. Chapman.

"More than 100," Rachel estimated.

When they were counted, there were 23.

Ms. Chapman follows-up on Soleap's comment. She is trying to build the students' sense of quantity, but realizes that to several students, 100 means "a lot." Rachel hasn't yet connected the fact that there are only 23 students and that each student brought one leaf into the room.

"What else do you notice about the leaves?" asked Ms. Chapman.

"They are different colors," said Pat.

"Some are yellow, and some are green, and some are red, and some are brown, and some are mixed-up," said Floyd.

Sorting by color is fairly difficult since several of the leaves are multi-colored. Ms. Chapman thinks it's important for the students to realize that identifying attributes and categories can be ambiguous; that defining the criteria for a particular group is sometimes arbitrary.

Sorting the leaves into piles by color took a while since there was some discussion about which pile particular leaves "belong in."

"Look at the pile of green leaves and the pile of yellow leaves," said Ms. Chapman. "Are there more green than yellow leaves, less green than yellow leaves, or are the amounts in each pile the same?"

Ms. Chapman wants the students to compare the quantities and be able to use the comparison words of more than and less than. Most students use "more" quite frequently; they are not as comfortable using the term, "less."

When the students had finished comparing the amounts in the two piles, Ms. Chapman asked, "What else do you notice about the leaves?"

"Some leaves are big and some leaves are small," said Leah.

"Which leaf do you think is the biggest?" Ms. Chapman asked.

Discussing which leaves are big and which are little is also ambiguous since some leaves are longer than others, while other leaves are wider, and still other seem to be larger in terms of area. This experience where students informally measure by comparing the lengths or widths of particular leaves helps lead to an intuitive sense of what it means to measure.

"Let's put the biggest one here, and the next biggest here," said Ms. Chapman. "Can we put all the leaves in order in a long line so the smallest leaf is at the end?"

This gives students the opportunity to see increasing/decreasing relationships (as in the number system). Ms. Chapman observed that while some students tried to put the leaves in order using random techniques, others began to compare each leaf as it was added to the series.

"Let's see how many different ways we can sort the leaves, put them in groups," asked Ms. Chapman.

"Some are spiky, " said Joshua.

Ms. Chapman gave the students opportunities to make their own observations and decide on their own groups.

Ms. Chapman asks her students to sort by different attributes so they develop an understanding that relationships among objects can change depending on the categorization. When the students grouped by "spikiness," the color of the leaves was ignored—some of the green leaves were with the brown and yellow leaves.

Vignette: Counting Noses in First Grade

In the vignette below, Mr. Peters informally assesses his first grade students' understanding of counting and their sense of quantity by asking them a simple question, and then by observing and probing their responses and interactions with one another.

The twenty-six first graders in Mr. Peters' class were into counting everything. One day in November, Mr. Peters asked the class, "How many noses do you think are in the class?"

Mr. Peters started the activity with a question he thought would be obvious to his students. He asked it anyway, wondering how the students would respond and what he would learn about his students' understanding of quantity.

Reuben grabbed his own nose, looked around the room and stated, "Lots! I think there are 100."

Reuben, who isn't consistently correctly counting ten objects, seems to equate amounts that are not immediately countable, with 100, which he considers a large number. His response is similar to many he has given in class.

"That's too many," said Soo; "There would have to be 100 people in the room to have 100 noses. There aren't 100 people here."

I know there are 26 noses," said Rico, "since there are 26 kids in our class."

Soo and Rico are always trying to make sense of amounts. Rico's response shows understanding of the notion of one-to-one correspondence—one nose per child.

"But Sandy and Frank and Sarah are sick today, so 26 is too many," stated Becky.

Becky is making further sense of the situation.

Several students stood up and tried to count noses. Mr. Peters suggested having one student go around to count noses. Antonio was selected.

Mr. Peters was surprised that so many students are trying to count. He knows that if he had asked the question, "There are 26 students, how many noses are there?" almost all would have quickly stated "26." But in a real situation, the students have a real need to check it out.

"One, two, three . . ., "Antonio counted, walking around the class, pointing at each student's nose. "There are 22 noses in the class," he announced.

Antonio was just gaining confidence in counting past twenty. He did the counting deliberately and seemed to take pride in doing it. Mr. Peters noticed that he counted noses rather than students.

"Antonio, you forgot to count yourself," said Rico, "that makes 23 noses."

"Don't forget Mr. Peters," said Soo, "he has a nose, too."

Rico and Soo had actively watched Antonio count noses, saying the numbers quietly to themselves. They were also thinking beyond, making further sense of the situation.

The next day the students counted the number of eyes in the room, again predicting before counting. The predictions ranged from 30 to 100. Several students predicted the actual count. To do the counting, the entire class stood up, and each child, in turn, counted on from the last child, sitting down afterward. On their turn, some students counted each eye separately, others just said the next multiple of two. After the first count, the class practiced counting by two's to again count the eyes. Students who were unsure of what number to say were encouraged to say the first number silently, and the second number aloud.

Counting different objects every day is part of Mr. Peters' ongoing plans for helping the students develop number sense. Students have been counting things daily since the beginning of school. Counting parts on the body will give the students opportunities to count by twos, fives, and tens, as well as by ones. He often chooses a different method for conducting the count. On this day, he asked all students to participate, each counting his/her eyes in turn. He had the students recount by twos and suggested a strategy for students who were unsure of how to do this. At other times, students will count objects by two's in unison.

On subsequent days, the class counted the number of fingers in the class (counting by 10's), the number of toes on the right foot (counting by 5's), etc. Students were asked to do counts where the number was the same as an earlier count, for example, to count the number of knees. For each count the students were asked to make predictions and to justify their predictions.

Mr. Peters knows that students need many experiences to develop an understanding of quantity and to develop fluency in counting. He will provide many opportunities for them to practice counting. Prior to counting objects he always asks students to make a prediction. Doing this daily, he gains much information about how his students are continually growing in their understanding of number sense.

Vignette: From Cans to a Third Grade Field Trip

Ms. Smith's third grade class is collecting empty cans to raise money for a field trip. Each day the students, working in small groups, figure out how much the cans collected are worth. On Monday, only 8 cans were collected. Ms. Smith asks the students to figure out how much the cans are worth, at 5 cents a can.

Some children add eight fives, other multiply eight by five, and others draw pictures of the cans, grouped in twos, and then count by tens.

Ms. Smith asks the students to explain to each other how they arrived at their totals. This process ensures that they justify their answers and helps them see that there is more than one strategy. Some will realize that some strategies are more efficient than others.

During the rest of the week, the following number of cans were collected: Tuesday, 13 cans; Wednesday, 12 cans; Thursday, 7 cans; and Friday 10 cans. Each day they find the amount of money earned.

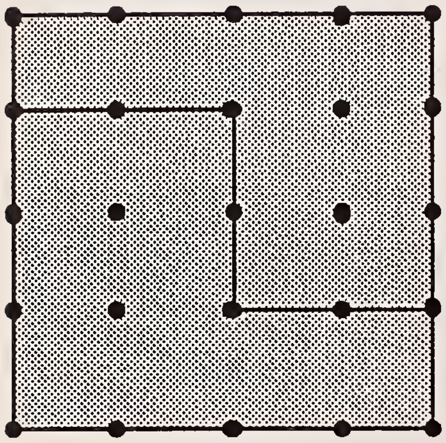
On Friday students are asked to the find the total amount for the entire week. Some students add all of the money and reach the total of 250 cents, or $2.50. A few students see that 50 cans were collected and at 5 cents a can would be $2.50. The class discusses how 250 can be thought of as 25 tens or as 250 ones, or as in money $2.50.

Using the data from the first week, the class predicts how much money they will earn in four weeks or during the entire school year. Would they collect $100 by the end of the year, if they continued to collect at the same rate as the first week?

Activity: Bay Khom, A Cambodian Counting Game

Bay Khom gives children opportunities for counting and for predicting. There are many other versions of this game from different countries throughout the world, such as the African game of Mancala.

Two children face each other. In front of each child are four bowls each with four stones or counters and to the right is a fifth bowl with five counters. The first child picks the counters from one of his or her bowls and counts one counter into each of the next four bowls. The child may choose in which direction to drop the stones (clockwise or counterclockwise) but must consistently go in the same direction throughout that turn. If there are counters in the next bowl after the child has placed all four counters he or she continues with placing the counters from the next bowl. If the next bowl is empty, the child takes the counters from the following bowl and keeps them. Then the other child has a turn. The child takes the counters from one of his or her own bowls and drops them, one at a time, into the next bowls until he or she picks up counters. The children continue to take turns until only one counter is left. The child with the most counters is the winner.



Dividing a geoboard into halves

Activity: Fractions on the Geoboard

Students work in pairs to explore fractional parts on the geoboard. Define the unit as the entire space inside when rubber bands are placed around the outside pegs. Ask students, "How many ways can you divide the geoboard into halves?" Encourage students to make halves by using rubber bands to form lines that are slanted or not in a straight line, such as a Z shape. Ask pairs of students to draw their halves on dot paper. They are to prove they have divided the board into halves by justifying their divisions.

When students are comfortable with halves repeat the activity by asking students to find fourths and eighths. Have students look at the relationships between fractions, e.g. one fourth equals two eighths.

For a culminating activity, have pairs divide a geoboard square into three or four parts. (Include halves, fourths, eighths, and or sixteenths.) Students record their square onto dot paper that is the same size as the geoboard. Make sixteen copies of each square design. Students then take their cutout sixteen squares, lay them out in a four by four array using any transformations to create a desired pattern. Glue the pattern onto oak tag and color. Students write about the pattern in terms of the fractional parts from which it was created.

Activity: Cambodian Counting

In Cambodia they use a counting system that is similar to a Chinese abacus (with five lower and two upper counters on each wire).

The counting sequence follows this pattern: *One, two, three, four, five, five-one, five-two, five-three, five-four, ten.* It continues, *Ten-one, ten-two, ten-three, ten-four, ten-five, ten-five-one, ten-five-two, ten-five-three, ten-five-four, twenty. Twenty-one,* etc.

Ask students to extend the pattern. This activity helps students understand there are different systems of counting in the world, and that the concept of counting is adding one more each time.

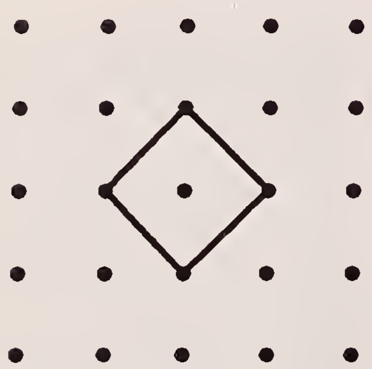
Money may be used to illustrate this kind of mathematical thinking. Use pennies, nickels, dimes. This could also be combined with introducing children to a Chinese abacus. It makes the ideas more concrete.

Activity: Number of the Day

For about five minutes each day, students find as many ways as they can to represent the date. For example, if the date is October 9, second grade students would find different expressions for the number nine, such as 12 – 3, 8 + 1, 2 + 3 + 4, etc. Third or fourth grade students might find expressions or equations for 109 (10/9) or 10,995 (10/9/95). This activity encourages each student to work at his or her own level. Students can record their work each day in a math journal. Sharing equations in the whole class each day can encourage students to use patterns and provide students with new ideas that they might use the next day. For example, observe what two different fourth grade students recorded for 109.

Sam		Bob
108 + 1 = 109	110 – 1 = 109	218 ÷ 2 = 109
107 + 2 = 109	111 – 2 = 109	327 ÷ 3 = 109
106 + 3 = 109	112 – 3 = 109	50 + 59 = 109
105 + 4 = 109	113 – 4 = 109	50 + 50 + 9 = 109
104 + 5 = 109	114 – 5 = 109	2 x 50 + 9 = 109
103 + 6 = 109	115 – 6 = 109	3 x 36 + 1 = 109
102 + 7 = 109	116 – 7 = 109	4 x 25 + 9 = 109
101 + 8 = 109	117 – 8 = 109	5 x 20 + 9 = 109
100 + 9 = 109	118 – 9 = 109	10 x 11 – 1 = 109

Grades 5-8: Number and Number Systems Vignettes and Activities



Vignette: Approximating Irrational Numbers

An easy way to introduce students to irrational numbers is by asking them to find the lengths of the sides of squares with known areas that they have constructed. In the vignette below, the students in Ms. Cho's sixth grade class have been making and finding areas of different squares on geoboards. (See the activity, "Area on the Geoboard" under Cluster III, Geometry and Measurement.) Ms. Cho uses this activity as a springboard for introducing irrational numbers. The students are discussing the following figure which has been drawn on a transparency on the overhead.

Ms. Cho: How did you figure the area of this square?

Judy: I saw that there were 4 half-squares inside, so the total is 2 whole squares.

Leon: I put another square around it that had an area of 4, and subtracted the four corner pieces, so that left an area of 2.

Ms. Cho: We all agree that the area of the square is two. Now I want to ask you a different question, how long is the length of a side of this square?

Tom: It's 1, because a side goes from 1 nail to the next nail.

Felipe: It can't be 1, because a diagonal is longer than a side.

Ms. Cho: Explain what you mean.

Felipe: (He goes to the overhead project and draws a 1 x 1 square on the transparency.)
See! This square has sides that are each 1 unit long. If I draw a diagonal from two opposite corners, it's longer than a side.

Sandy: That diagonal Felipe drew, is the same length as the side of the square we're looking at.

Ms. Cho: So how many units long do you think it is?

Tom: It must be $1\frac{1}{2}$ since it's less than 2.

Ms. Cho: How could we find out for sure?

Tom: We could measure it.

Ms. Cho: That would be one way. We don't, however, have a ruler that is calibrated in geoboard units. Is there another way we could figure it? What do we know about the relationship between the length of a side of a square and its area?

Fran: Multiply the length of a side by the same number and you get the area.

Rafael: The formula is A equals side squared.

Ms. Cho writes the two formulas, $A = s^2$ and $A = s \times s$, on the board.

Ms. Cho: If the area of our square is 2, how could you find the length of a side?

Sandy: Find a number that when multiplied by itself equals 2.

Ms. Cho: Tom suggested $1\frac{1}{2}$. Use your calculators to see what $1\frac{1}{2}$ squared is.

Most of the students enter "1.5 x 1.5 =" on their calculators. Some remember that a shorter way is to enter "1.5 x =". A couple of students have calculators with a square key and know how to use it. A few students need to be reminded that $1\frac{1}{2}$ is the same as 1.5.)

Ms. Cho: Did 1.5 x 1.5 equal 2?

Stan: No, it was 2.25. That's too big.

Ms. Cho: What's another estimate for the length of the side?

Leon: I think $1\frac{1}{4}$. That's between 1 and $1\frac{1}{2}$.

Judy: Yes, but 2.25 is only a quarter away from 2, and 1 is a whole unit away from 2, so the answer must be closer to 1.5 than to 1.

Felipe: I'm going to try 1.3

Ms. Cho: OK, let's try both those numbers. (pause) What did you get?

Leon: 1.25 time 1.25 is 1.5625. That's too small.

Felipe: 1.3 times 1.3 is 1.69. 1.3 is too small too.

Fran: It's bigger than 1.3. Maybe it's 1.4.

Joe: 1.4 times 1.4 is 1.96. That is real close. It's going to be just a little more than 1.4.. Maybe it's 1.5.

Susan: No! That's what we tried first!

Joe: Well, then it doesn't work because 1.5 is the next number after 1.4

Judy: Not really. There are some in between, like 1.41 and 1.42, and 1.43, and...

Joe: Oh, yeah. Well, maybe it's 1.41.

Leon: 1.41 times 1.41 is 1.9881. Now it's never going to get there! It's just like those division problems that go on and on.

Ms. Cho: Let's see what the rest of you think. Is 1.41 squared as close as you can get to 2? In the next few minutes, see if you can get closer to the number 2 by squaring a number. You may choose to work by yourself, with a partner, or with your group of four. Please keep a record of each number you try and its square. Then we'll share what we find out.

Activity: How Much is a Million?

Getting a sense of large quantities can be motivating and engaging for students in grades 5–8, particularly if the students investigate questions they find interesting. You might start by asking the students a question such as: If you earned one million dollars a year, how much money would you have earned per day? Ask students to guess, and then ask them to share strategies about how they would find out. If your class immediately says that you would divide one million by 365 on the calculator, you might discuss whether a person earns money daily or if the calculations should be based on number of days a person works. Some students may be interested in finding hourly rates for this salary.

Identify and investigate other situations involving one million and have students explain each situation in terms that can be readily comprehended. For example, students want to find out how long it would take to count to one million. They might define the situation by stating they would count one number per second, 24 hours per day. Students need to determine whether it makes more sense to report the answer in hours, days, weeks, etc.

Other situations that students might find interesting include the following:

- How high would a stack of one million $1 bills be?
- How old is a person who is one million minutes old? one million hours old? one million days old?
- Assuming that Fenway Park (capacity 34,182) is full each game, how many home games would the Boston Red Sox have to play before they have played in front of one million people? (Does it matter that many of the fans attend each game?)
- How much space would it take to have a standing crowd of one million people?
- What size box would hold one million centimeter cubes?

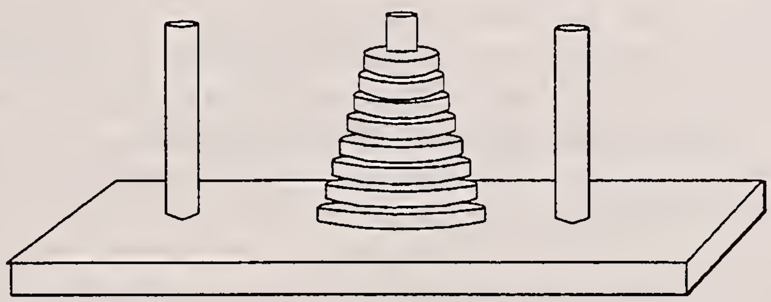
Ask students to create their own situations that involves one million and to explain each situation in terms that can readily be comprehended.

Grades 9-12: Number and Number Systems Vignettes and Activities

Vignette: Tower of Hanoi

Mrs. Higuera is experimenting with having her students work in groups, and today puts them into random groups of three to investigate a classic mathematics problem.

In a temple in Benares, India, according to legend, under a dome that marks the center of the world, there is a Tower of Hanoi, with 64 disks that priests move day and night. When they have moved all the disks, the universe will end.



Each group is given three posts, with eight graduated disks on the center post. The question Mrs. Higuera poses is, "Is it possible to move the disks to one of the other posts given the restrictions that disks are to be moved one at a time and that no disk may be placed on a disk of smaller diameter?"

As Mrs. Higuera observes her class she sees some student behaviors which she thinks of as predictable, and others which she finds surprising. She notices that despite their previous class discussions of the importance of everyone contributing to the group effort, Bob has his head in his hands, is not participating in his group, but is apparently watching someone in another group. His two partners seem content to be ignored by him and are engaged in the problem. Similarly she is not surprised to see that Josephine has taken over in her group, confidently moving the disks herself, asking her two partners to take notes. She is disappointed to see, however, that Bill is not participating in his group at all. He is bent over writing, trying to solve the problem by himself, using paper and pencil. He performs well in her class and she wonders if he will be able to solve the problem without using any concrete materials. She is not surprised to see him working on the problem. She is surprised, however, that a student who is usually so cooperative has completely ignored her instructions to work with his partners. Mrs. Higuera is also surprised to observe Gladys. Her group has decided to take turns moving the disks and Gladys, who never does the assigned written work, is not only participating when it is her turn but is also making helpful suggestions to her two partners.

Mrs. Higuera reflects on what other teachers in her cooperative learning support group have reported: it will take time for students to learn to work together; it won't necessarily work well the first time you put students in groups, so you will need to persist; and if you observe carefully, you will learn things about your students which were not previously visible in the classroom.

Mrs. Higuera also notices that some groups continue to use what appears to be a trial and error strategy. Some groups are persisting in the task with high energy but others appear to be getting discouraged.

Interestingly, Gladys' group is the one group that has apparently remembered an earlier class discussion on problem solving strategies and is trying one of the suggested approaches: "try an easier version of the problem." They are trying to solve the problem using only three disks. As Mrs. Higuera approaches their table, she notices that Herbert, who has become the note taker for the group, has written, "one is easy," and, on the next line, "two is just as easy." Mrs. Higuera decides to invite groups to report on their ideas, and calls on Gladys when it is time for her group to report.

Mrs. Higuera is surprised that no group has been successful in moving all eight disks. Some groups have picked up on the "easier version" strategy and have been successful with four or five disks but are not yet able to apply their successful method to more disks. She thought high school students would have no difficulty generalizing from four or five to eight disks. She was delighted, however, that two groups noticed that moves for the odd and even disks alternate - from left to right, or clockwise to counterclockwise if arranged in a circle. One of her goals is to help her students develop their problem solving skills. She listened as Billy explained, "Well, I did two and that took three moves. Now I'm going to do three, I have to move the top two, then move the bottom one, and then put the top two back on top. So that's three and one and three. It's going to be seven moves." She is impressed, both with his reasoning, and with his communication with his partners.

# of disks	# of moves
1	1
2	3
3	7
4	14

In another group she notices that they have started to make a chart. She wonders if they will find a pattern, or if their miscounting (14 for 15) will lead them astray.

She thinks about her plans and expectations for the next few days—students solve the original challenge, introduce the legend, ask the students to use the legend to estimate the end of the universe—perhaps by generating the formula for the number of moves, perhaps by timing themselves with one disk, two disks, three disks, ...and writing a formula for the time required for 64 disks. She realizes that this may take more time than she anticipated—and again reflects on

the discussions in her teacher support group, "When students are more involved, when they are really figuring things out for themselves, you may feel you are covering too little content." She wonders, "Are the students really learning more this way?" Although she is trying to change the way she assesses her students' progress and achievement, she still finds herself thinking, "It may take several days to get to the formula. It will take time to get to a discussion of recursion." She expects that they will generate

$$f(x) = 2^n - 1$$

and

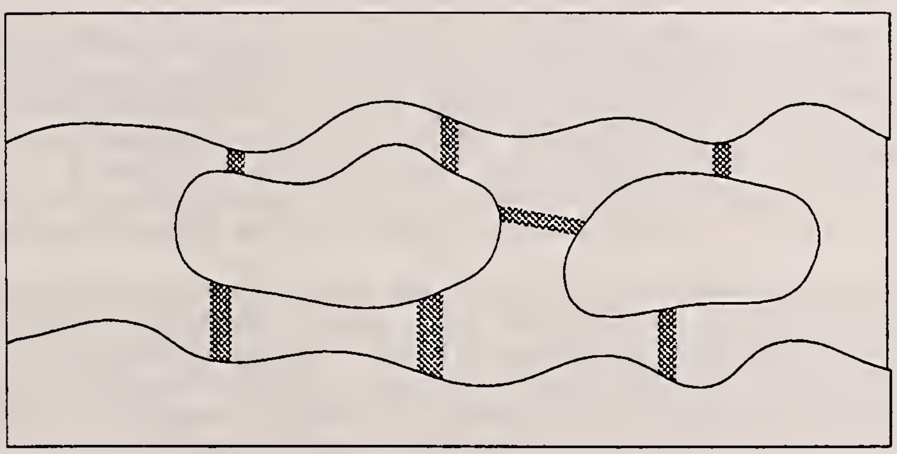
$$g(1) = 1$$
$$g(n + 1) = 2\,g(n) + 1$$

and be interested in comparing them.

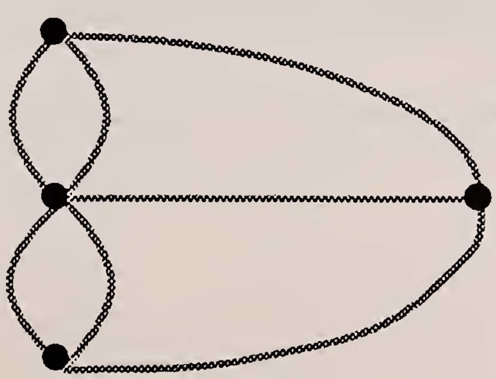
Activity: Bridges of Konigsberg

Ms. Ayala decided this year to introduce her junior students to graph theory. She began the unit with the classic problem of the bridges of Konigsberg.

In the eighteenth-century Prussian town of Konigsberg, there were seven bridges connecting the city to two islands. Residents of the city often took evening walks from one section of the city to another, traveling over these bridges. They began to wonder whether one could walk throughout the city, crossing each bridge exactly once.



This situation was modeled by Leonard Euler (1707-1783) with a graph similar to the one below in which the vertices denote land areas and the edges denote bridges.



Vignette: Planning Construction

Ms. Ayala also decided to try to introduce graph theory in a way which will help her students see that graph theory can be applied, by them and in the world outside school. She knows that in her vocational school each year her students carry out a building project. She talked with the trades department and found out that this year the project will be to plan and build a gardener's tool and work shed. Working with the trades department, Ms. Ayala worked out a plan.

She began the unit by asking her class to list the tasks included in the construction. The students generated the lists in small groups and then put them together to form a class list. The beginning list compiled by the class included:

- buy the wood
- buy the electrical cable
- buy the cement for the foundation
- buy nails
- buy a sink
- buy pipe
- buy shingles for the roof
- pour the foundation
- put in the sink
- connect the sink to the city sewer
- frame the shed
- put on the roof
- put in shelves
- make places to hang tools
- put up the walls
- put in the lights and electrical outlets

Ms. Ayala next organized the students into groups, with each group assigned one set of tasks: foundation, walls, plumbing, storage and work spaces, roof, doors and windows, Students were asked to talk with expert sources (other teachers, seniors who had done a similar project the previous year, gardeners, the owner and employees of a local construction company) in order to develop more complete lists for each subset of tasks. Following that, students worked in class to develop draft versions of directed graphs. The group working on storage and work spaces developed the graph at upper right.

Students were then asked to return to their expert sources to verify the graphs and to estimate the time for each task. When the individual graphs were finished, they combined them to make a graph showing how they related to one another, and then used them to determine a "window" for each task. If the shed is to be completed by May 1, what is the earliest date, and what is the latest date, that each task could be begun?

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Later, Ms. Ayala's students studied other topics from graph theory, including trees and minimal paths.

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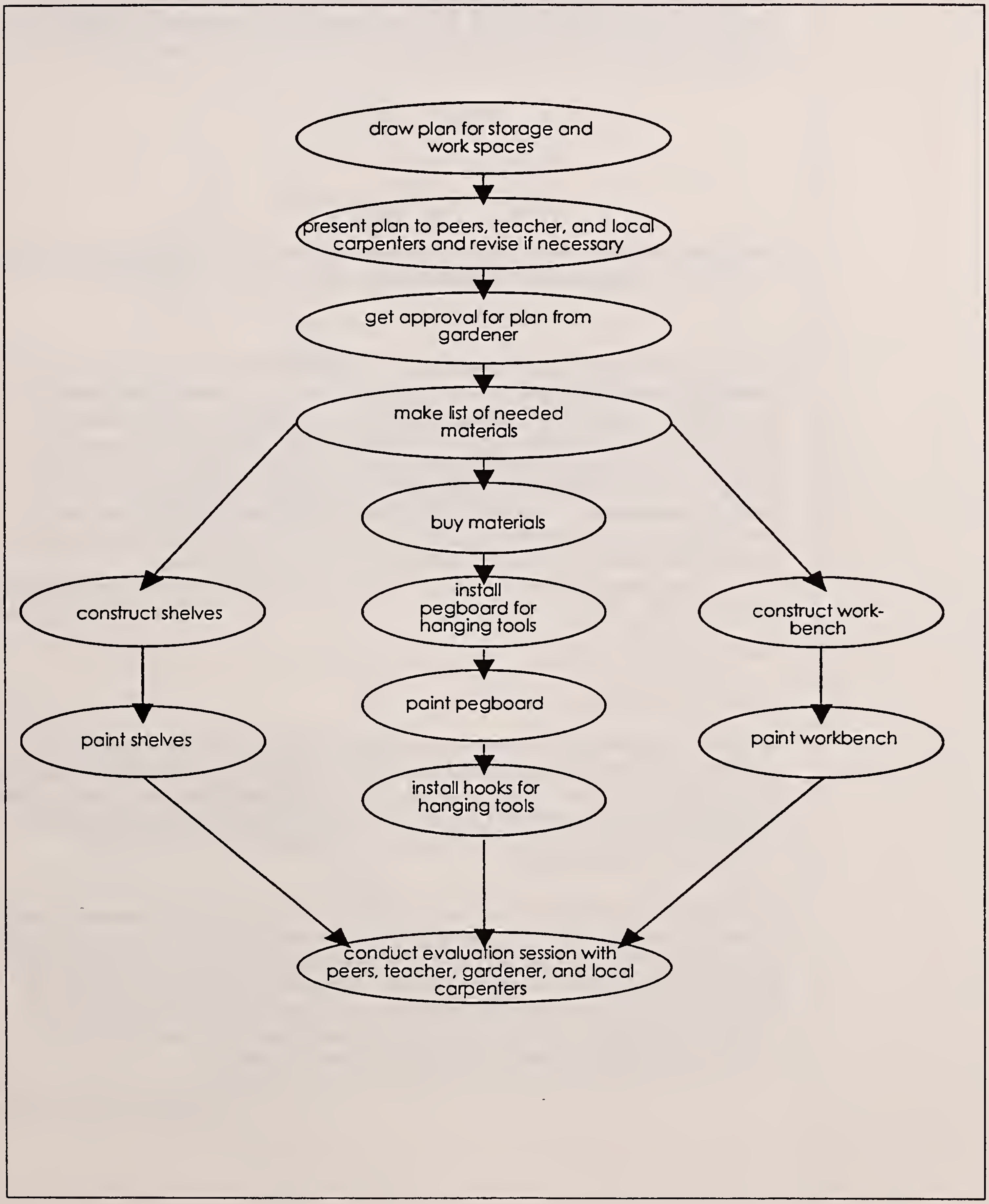


are challenging even for students with strong mathematics backgrounds. This topic also provides teachers with an excellent opportunity to help students make connections within mathematics and to other subjects.

Mrs. Reyes gives her students the following problem.

There are two athletic stores in a small rural town. Surveys show that each year 85% of the people who shopped at store A at the beginning of the year are still shopping at store A at the end of the year, and that 75% of the people who shopped at store B at the beginning of the year are still shopping at store B at the end of the year. This trend can be represented by a transition matrix, T:

$$\begin{array}{ccc} & A & B \\ A & .85 & .15 \\ B & .25 & .75 \end{array}$$

Mrs. Reyes explains that if the stores A and B initially had 45% and 55% of all customers, respectively, this can also be represented by a matrix, G:

$$G = .45 \quad .55$$

She then asks her students some direct questions, "What proportion of the sales will each store have after one year? After two years? After three years?" and a more open-ended question, "If you assume this goes on indefinitely, what do you think will happen in the long run?" Students share their tentative ideas, "B starts ahead but A is going to gain over B," "A is going to keep gaining," "It might get to be 50-50," "It won't stop at 50-50, it will be a monopoly," "B is going to go out of business." Mrs. Reyes then encourages her students to use calculators and work with a partner and to pursue their initial ideas by carrying out some computations.

$G \times T = (.52 \quad .48)$

$G \times T \times T = (.562 \quad .438)$

$G \times T \times T = (.5872 \quad .4128)$

As the students begin to work, Mrs. Reyes circulates around the room to probe their thinking and to pose additional challenging questions: "How long do you think it will take for the store which is gaining sales each year to have a monopoly on shoe sales in the town?" She is also watching to see if any students try to find more efficient ways of calculating, for example: $T^2 \times T^2 = T^4$; $T^4 \times T^4 = T^8$; *etc.*

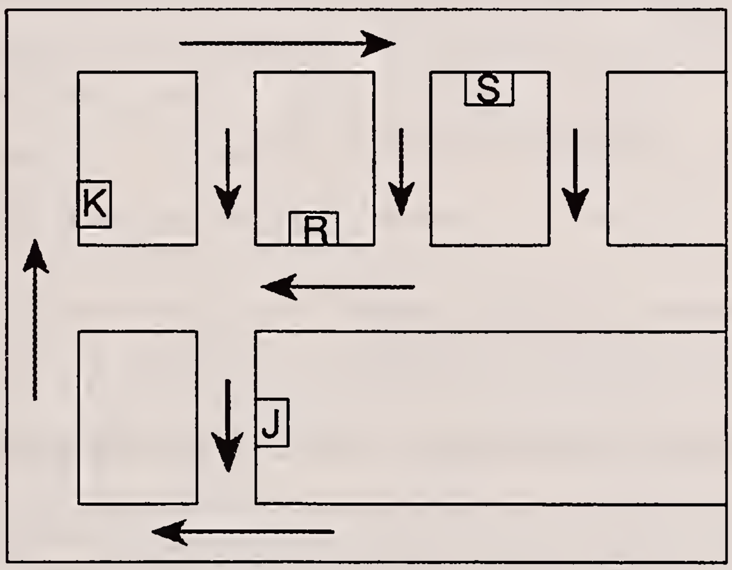
She knows that students may be surprised to realize that the problem defines a steady state, and that the steady state for this example is: each year .625 of the population will be regular customers of store A and .375 of the population will be regular customers of store B, represented by the matrix (.625 .375). She thinks that experiences with examples such as this, which run counter to intuition, will help her students develop an understanding of the need for testing one's ideas, and of the need for proof.

Activity: Directed Networks

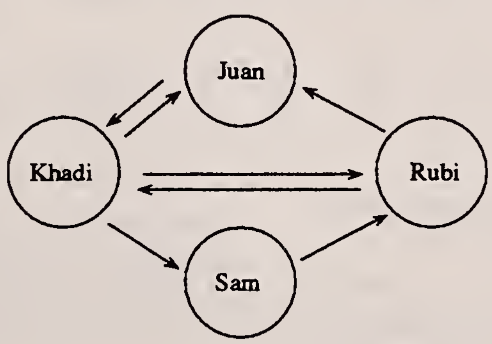
Matrices may also be used to investigate directed networks. Students might be given the map below, with the accompanying explanation.

Suppose Juan, Khadi, Rubi, and Sam live in a neighborhood with several one-way streets. The map shows the location of their homes.

The students might then be asked to develop directed graphs that show which homes Juan, Khadi, Rubi, and Sam can drive to directly, without going



past the home of another of the group. A student might draw, for example:



Next, students would be asked to write a matrix, A, to represent these relationships:

	J	K	R	S
J	0	1	0	0
K	1	0	1	1
R	1	1	0	0
S	0	0	1	0

Students might also be asked to use calculators to find A^2 and A^3 and interpret their meaning.

Ask students to assign a different number to each letter of the alphabet. They might, for example, set:

A = 1, B = 2, C = 3, D = 4, ...

Then ask them to put a message in the form of a 2 x n matrix, such as:

$$\begin{matrix} P & E & S & H & L \\ L & A & E & E & P \end{matrix}$$

Ask them to code the message using the letter/number correspondence to form a matrix, M:

$$\begin{matrix} 16 & 5 & 19 & 8 & 12 \\ 12 & 1 & 5 & 5 & 16 \end{matrix}$$

Then ask them to choose a 2 x 2 matrix, C, which has an inverse. For example:

$$C = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$$

They can then recode the message by multiplying M by C, using calculators to do so:

$$CM = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix} \quad \begin{matrix} 16 & 5 & 19 & 8 & 12 \\ 12 & 1 & 5 & 5 & 16 \end{matrix}$$

$$= \begin{matrix} 40 & 7 & 29 & 18 & 44 \\ 96 & 19 & 77 & 44 & 100 \end{matrix}$$

After exchanging messages and coding matrices, students can decode the messages by multiplying by the inverse of C:

$$\begin{matrix} -2 & 1 \\ 3/2 & -1/2 \end{matrix} \quad \begin{matrix} 40 & 7 & 29 & 18 & 44 \\ 96 & 19 & 77 & 44 & 100 \end{matrix}$$

$$= \begin{matrix} 16 & 5 & 19 & 8 & 12 \\ 12 & 1 & 5 & 5 & 16 \end{matrix}$$

Vignette: Mathematical Structures

Ms. Israel wants to show a connection between a geometric concept and an algebraic concept. She also wants to introduce the concept of structure within two systems in mathematics.

Ms. Israel begins with a hands-on activity. She hands out, to each student, a sheet of paper with a square on it labeled ABCD and a cardboard square of the same size, also labeled ABCD.

Ms. Israel: Today we're going to experiment with rotating a square. Place your cardboard square on the square on the paper and line up the corresponding parts. Try to find a away to rotate the cardboard square so it fits on the paper square.

Charles: If you rotate it by 360 degrees it will return to its original place.

Ms. Israel:	Today we're going to experiment with rotating a square. Place your cardboard square on the square on the paper and line up the corresponding parts. Try to find a away to rotate the cardboard square so it fits on the paper square.
Charles:	If you rotate it by 360 degrees it will return to its original place.
Ms. Israel:	Good. Is there any other way to do it?
Aisha:	How about 90 degrees?
Ms. Israel:	Let's try it. Does it fit on the square on the paper?
Nikisha:	Yes, it's sitting on top of the other one, but Charles turned his the wrong way.
Ms. Israel:	Charles, what happened when you turned yours the other way?
Charles:	It works either way you do it.
Ms. Israel:	Good observation, As a class, lets's agree whether 90 degrees will mean 90 degrees clockwise, or 90 degrees counterclockwise. Aisha, you suggested 90 degrees, which will it be?
Aisha:	90 degrees clockwise, that's the way I turned mine.
Ms. Israel:	Fine. Some of you may want, later, to experiment by deciding that 90 degrees means 90 degrees counterclockwise. Let us know what you notice when you do that. Is there another way to rotate the square so that it sits on top of itself again?"
Aisha:	Yes, 180 degrees.
Ms. Israel:	Good. Does anyone see a pattern?
Nikisha:	I see! You just multiply 90 by one, two, three, and four and you have four answers. Maybe it's because a square has four sides. The last answer is 90 x 4 = 360.
Ms. Israel:	Excellent.

Ms. Israel then writes on the board:

j = a rotation of 90 degrees
k = a rotation of 180 degrees
m = a rotation of 270 degrees
n = a rotation of 360 degrees

Together, Ms. Israel and the students complete the following chart.

*	j	k	m	n
j	k	m	n	j
k	m	n	j	k
m	n	j	k	m
n	j	k	m	n

Ms. Israel then continues the class discussion: Earlier this year we discussed systems and some of the characteristics of systems. What are some of the things you remember about the systems we studied?

Nikisha: Sometimes they are commutative, sometimes they are not.

Charles: Usually there is an identify element, like one for multiplication or zero for addition.

Aisha: There are inverses too, like plus two and minus two add up to zero.

At the close of the discussion, Ms. Israel explains the homework. She asks the students to study the chart they have just developed, and to write about what they notice about the system. She suggests that they consider some of the properties they have mentioned:

Is it commutative?

Is there an identity element?

Does each element have an inverse?

She also asks them to complete a chart to show multiplication of the set of numbers, i, i^2, i^3, i^4 in the complex number system. Tomorrow they will talk about the similarities between these two systems and compare them.

x	i	i^2	i^3	i^4
i				
i^2				
i^3				
i^4				

She anticipates that all of the students will notice similarities between the two systems and that at least some of the students will notice that they are isomorphic.

An Adult Learner in a pre-GED class wrote:

"We learned today how to light a light bulb by making a complete circuit ... you need direct current (DC) to make it happen. We also learned some math. If one battery has 1.5 volts and you tape 25 of them together ... to see if the bulb burns brighter (it does), you can figure it out in your head that you are using 37.5 volts of power to do it. All you do is multiply 25 times one to equal 25, and then cut that figure in half since .5 means half. 25 divided by two equals 12.5. Add it all together and you get 37.5."

B. Estimation and Computation

> *The purpose of computation is to solve problems. Thus, although computation is important in mathematics and in daily life, our technological age requires us to rethink how computation is done today. Almost all complex computation today is done by calculators and computers. In many daily situations, answers are computed mentally or estimates are sufficient, and paper-and-pencil algorithms are useful when the computation is reasonably straightforward.... Clearly, paper-and-pencil computation cannot continue to dominate the curriculum or there will be insufficient time for children to learn other, more important mathematics they need now and in the future.*
>
> NCTM Standards, p.44

Children's natural use of counting leads to the need to use the operations of addition, subtraction, multiplication, and division in more complex situations. Learning how and when to perform operations should always make sense to students. Students need to encounter a variety of situations to develop a sense of when the operations would be useful. Mathematical operations must be understood if they are to be useful to students.

Ongoing estimation experiences in school and outside it, help promote students' number and spatial sense, and sense of judgment. Experiences estimating help students develop insights into concepts and procedures, flexibility in working with number and measurement, an awareness of what are reasonable results. Students need to be able to estimate quantities and measures. For example, About how many people are in the room? in the US.? What is the approximate length of the room? Approximately how many miles is it from the east coast to the west coast? Students also need to approximate computational answers. For example, The cost of the bread is $1.69, the milk is $2.07, and the eggs, $1.89; about what will the total cost? Students need to know what is meant by an estimate, when it is appropriate to estimate, how close an estimate is needed in a given situation, and explain why an estimate is reasonable.

While calculators and computers are used to do most of the complex computations in today's world, the ability to estimate is critical. In some situations making estimates is sufficient, for example, in trying to figure out it there is enough time to get to a theater before the performance starts. Estimates may be used prior to making an exact calculation, for example, estimating the cost of items in a grocery cart prior to seeing the total on the cash register. Estimates are also used to evaluate the reasonableness of answers

produced by calculators and computers; it is easy to push a wrong button or to enter the wrong number by mistake. Students need to decide whether it makes sense to use paper-and-pencil, a calculator, or to compute mentally for a given situation.

Computation is important; it is important for students to be able to find answers to problems involving quantities. Developing paper-and-pencil computational proficiency, however, is no longer the primary goal of elementary mathematics programs. As students solve more actual real problems, they all find estimation and the use of calculators to be valuable. How should students learn computational procedures? It is not always necessary to "teach" specific procedures for students to figure out an answer. Students can often informally figure out answers to problems before encountering the formal mathematical representations. Young students, for example, can figure out how to share a dozen baseball cards among three friends before being introduced to formal multiplication or division. Older students can find several ways to find the square root in order to calculate the length of the side of a square with area of 5. Through sharing their individual approaches and strategies, students realize there are many ways of looking at and solving a problem, and that some ways are more effective and/or efficient than others in particular circumstances.

Students need to be fairly efficient when finding numerical answers to problems, even if they don't always have a calculator. That does not mean, however, that they all need to learn the traditional standard computational algorithms. Students can invent procedures that are sensible and efficient. A fourth grade student, for example, might find the difference between 187 and 536 by mentally adding on 13 to make 200, adding 300 to make 500, and finally adding the 13 to 36 to get an answer of 349. Students can be shown how to do computational procedures, and if they make sense to them, they will use them. Students should not be prohibited from using their own methods if they are accurate and make sense to them.

It is important for students to know the basic mathematical facts, such as the product of 6 times 9. It is equally important for students to understand the meanings of the mathematical operations and how learning facts is useful to them. Students should be encouraged to learn the facts in a variety of ways, without the pressure of being expected to recall facts instantaneously.

In ABE math and ASE/GED math, the teaching of estimation to adults takes on a meaning beyond "rounding off" numbers as it is currently presented in many adult basic education texts. The math curriculum should include ways for the adult learner to develop an estimation mind-set when approaching problem-solving in real life contexts and in test taking situations such as the GED. Instructors should be especially aware of the educational backgrounds of ESL students when teaching estimation (or other number skills), since many learners come from cultures where decimals rather than fractions are used for computing. In the workplace education setting, it is important that the learner have a solid understanding of estimation in order to check the reasonableness of results, since on the job

I asked a group of my adult GED math students to tell me how much it would cost if I bought four shirts for $7.98 each. They were told they could figure it out any way they wanted, except they could not use paper and pencil. I watched as they used their fingers in the air or "wrote" on the desk. Most were able to multiply and get the right answer. When I asked "how" they got their answer, all agreed that they needed to multiply $7.98 by four.

I the asked if they were in a store and had to figure out the same problem would they have done it the same way. All agreed they probably would not solve it the same way in "real life." Some said they would have multiplied four by seven plus four by one and then subtracted eight cents from that total. Others said that they would have rounded $7.98 to $8.00, multiplied that by four and then subtracted $.08 from the product. I then asked why no one admitted to solving the problem like that in class. The response was that this was math class so they needed to do it out."

--Marilyn Moses, GED teacher, Brockton Adult Learning Center

NCTM Estimation and Computation Standards

K-4 Standards

Standard 5: Estimation
In grades K-4, the curriculum should include estimation so students can—

- explore estimation strategies;
- recognize when an estimate is appropriate;
- determine the reasonableness of results;
- apply estimation in working with quantities, measurement, computation, and problem solving.

Standard 8: Whole Number Computation
In grades K-4 the mathematics curriculum should develop whole number computation so that students can—

- model, explain, and develop reasonable proficiency with basic facts and algorithms;
- use a variety of mental computation and estimation techniques;
- use calculators in appropriate computational situations;
- select and use computation techniques appropriate to specific problems and determine whether the results are reasonable.

5-8 Standards

Standard 7: Computation and Estimation
In grades 5-8, the mathematics curriculum should develop the concepts underlying computation and estimation in various contexts so that students can—

- compute with whole numbers, fractions, decimals, integers, and rational numbers;
- develop, analyze, and explain procedures for computation and techniques for estimation;
- develop, analyze and explain methods for solving proportions;
- select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods;
- use computation, estimation, and proportions to solve problems;
- use estimation to check the reasonableness of results.

Note: Estimation skills and the use of calculators are necessary components of mathematics curriculum in grades 9-12.

ABE Standard 5: Estimation
In the adult basic education classroom, curriculum design must include approaches to teaching estimation which allow the learner to:

- explore and develop the concepts underlying a variety of estimation techniques and strategies for whole numbers, fractions, decimals, and percents;
- recognize when as estimate is appropriate and useful in real-life situations and the role estimation plays in adult life;
- apply estimation techniques in working with quantities, measurement, computation, problem solving, and in workplace and test situations;
- use estimation to check the reasonableness of results.

Grades K-4: Estimation and Computation Activities

Activities	Rationales	
Literature Connection: *How Big is a Foot?* by Rolf Myller Estimate the number of connecting cubes in a bucket by showing what a unit of ten looks like. Compare the estimate to the actual number by stacking the cubes into towers of ten and counting.	From the earliest mathematical experiences, students need to have a sense of terms such as about, near, more than, less than, etc. By encouraging students to estimate, they accept estimation as mathematics and learn that math does not always lead to an exact number.	Activity: As Big as My Foot
	Use of manipulatives helps children develop conceptual understanding of estimation. A referent set (sample) helps children make estimates within a reasonable range. Place value and number skills are reinforced as students place estimated items into groups that facilitate counting.	Activity: Estimate and Count
Write a simple arithmetic expression on the board (i.e. 12 + 25) Ask students to compute the answer mentally, then share their mental math strategies.	Students need to have multiple experiences with basic fact strategies to encourage them to be flexible in their thinking and mental computation. Being able to communicate one's strategy displays understanding.	Activity: Mental Computation
Work in groups to determine how $10 can be spent. The group must buy at least three items, may buy more than one of an item, and should spend as much of the $10 as possible. Use estimation to show the approximate total cost of the items purchased. Literature Connection: *Alexander Who Used to be Rich Last Sunday* by Judith Viorst	Cooperative groups encourage communication in mathematics.	Activity: Spend $10
Estimate the distance your class travels within the school in one day. Use a trundle wheel, pedometer, yardstick, or tape measure and calculator to measure the actual distance. What is the distance in feet? Inches? Yards? Repeat on other days. On which day of the week does the class travel the		

Activity: Distance in School	Estimate the distance your class travels within the school in one day. Use a trundle wheel, pedometer, yardstick, or tape measure and calculator to measure the actual distance. What is the distance in feet? Inches? Yards? Repeat on other days. On which day of the week does the class travel the greatest distance? The shortest distance? Compare your measurements with other classes. How far in a week, month, or year does the entire class travel?	Estimation can be integrated with measurement. The appropriate use of technology as a tool can also be integrated. (Technology can make solving complex problems easier.)
Activity: How Many Grains?	Estimate the number of grains of rice in a container by choosing tools from a set of materials (balance scales, weights, rulers, calculators, graph paper and measuring cups). Develop two different strategies to arrive at solutions. Compare results. Literature Connection: *A Grain of Rice* by Helena Clare Pittman	Students need experiences with estimating larger quantities. These experiences may lend themselves to developing multiple strategies that may result in multiple solutions. Groups should present strategies. Often times the exact number is not the goal, rather a reasonable range is. When estimating larger numbers, it is helpful to use a referent (smaller sample) to help determine reasonableness of results.
Activity: Wasted Water	If there is a leaky faucet in the school, pose these questions. About how much water is wasted per hour? Per day? (This could be extended to include discussion of science and social issues, cost, etc.) Allow cooperative groups to attack the problem. Have each group report their solution and strategies for collecting the data and solving the problem. Have a MassSave representative talk to the class about water conservation.	Engaging problems can be relevant and integrate other subjects. Compelling problems may take more than one day to complete, and could spark new questions to be solved.
Activity: Fund-raising Math	Use a fund-raising project or a simulation to develop computational skills as they arise naturally. For example, a candy sale might involve determining costs of supplies, selling price, profit or loss, and sales goals. Students could keep accounting records, and make graphs of daily sales, most popular items, and top salespersons.	Activities such as this encourage the use of estimation, mental math strategies, and computational skills. Students could utilize calculators and computer software, such as spreadsheets, databases, and graphing programs.

Grades 5-8: Estimation and Computation Vignette and Activities

Vignette: Popcorn Estimation

Mr. Thomas became intrigued after reading a MEAP report about performance assessments that were piloted in Massachusetts during the spring of 1989. How would some of his eighth grade math students respond to the same question?

He gathered up the materials he needed: popcorn kernels in their original container, a scoop, a set of different sized containers marked in milliliters with the smallest a 30 medicine cup, pan balance, and a set of weights totalling 120 grams.

Sasha and Andria were the first pair of students he called into the conference room. He tried to make them comfortable by telling them that he wanted to learn more about how eighth graders thought about and solved problems. He was going to give them a problem to solve, but the problem wasn't going to affect their grades. He wasn't going to help them solve the problem, he just was going to observe what the two of them did. He wanted to hear their thinking, since that would help him understand his students better and, hopefully, become a better math teacher.

Mr. Thomas stated the problem. "You are to estimate the number of popcorn kernels that are in the container. You may use any of the materials on the table to help you do this. The only thing that you can't do is count all the kernels in the container. The closest estimate to the real number would be the best estimate for me."

Sasha put 1 kernel in the pan balance, and a 1 gram weight in the other side, which immediately went down. "One is too light," said Andria. "We need to put more kernels in the pan. Let's put in 25."

"Twenty-five is 2 grams," said Sasha, "and we've hardly weighed any of the popcorn. Let's try weighing it all."

The two girls poured the popcorn into the pan which filled up before the container was emptied. All the weights were not enough to balance it. They then poured all the kernels into the largest container and observed that there was about 800 ml.

"Hmm," said Andria. "Let's see. If we take half of the popcorn that would be 400 ml, but that's still to heavy to weigh, and then half of that would be 200 ml, or a quarter of the popcorn."

"That's still too heavy," said Sasha after putting all the weights in the balance. "We're going to have to halve it again."

The 100 ml weighed 73 grams. The girls multiplied this by 8 to get a total weight of 584 grams for all the kernels. "All the popcorn weighs 584 grams," they stated in unison.

"Yes, that makes sense," said Mr. Thomas. You found the weight of the popcorn, but I asked you to estimate how many kernels there are. Can you use what you've found out so far, to find this?"

This vignette is based on a MEAP report, "Popcorn Estimation," Beyond Paper & Pencil, MA Department of Education. (January 1990).

"I forgot about that," said Sasha. "Well, we found out that 25 kernels weighed 2 grams. If we divide 2 grams by 25 kernels," she said while pushing the keys on her calculator, "that comes out to be equal to 0.08 grams per kernel."

She wrote on the paper, $\frac{k}{g} = \frac{1}{.08} = \frac{x}{584}$

She used her calculator to divide 584 by .08. "Our estimate is 7300!" she stated triumphantly.

After they left the room, Mr. Thomas thought a moment about their work. At first they did not have a plan for tackling the problem. Their approach of successively cutting the quantity of popcorn in half, made sense, and was an approach he never would have thought of. While they lost track of the original question in the middle of the problem, when he reminded them of it, the girls were able to connect their thinking to a rather formal strategy—they set up a proportion equation and then solved it.

Sandy and Mickey came in next. After hearing the instructions, Sandy took the 100 ml container, and started counting kernels, as he dropped them in, one by one. ". . . 179, 180, 181, . . " he counted.

"Hurry up, we're going to be here all day," said Mickey. "That container's too big. Let's pour them into this one, it will fill up faster." They poured the popcorn into the 50 ml container, and finished filling it. It took 416 kernels. They then poured the rest of the kernels into the large container, which filled to the 700 ml mark.

"Now we divide 700 by 416," said Sandy.

"No, we divide 700 by 50," said Mickey. "That will tell us how many of these [small containers] fit in here [the large container]." He did the division using a calculator and said the answer was 14.

Sandy used his calculator to divide 700 by 416, which gave an answer of 1.6.

Working independently, Mickey multiplied 416 by 416, and got an answer of 5824. "This answer can't be right," he said. "It's too low. There's got to be more than 5824 popcorn kernels there."

When asked by Mr. Thomas to explain what he was doing, Sandy said, "I'm supposed to divide 700 by 416 and then multiply that answer by 50, I think." He didn't do the calculations, however.

Mr. Thomas thought about how diverse the students were in his class. He wondered if Sandy could have made sense of the problem, if he had proceeded on his own. He seemed to be doing the calculations simply because they seemed to be expected, rather than because there was a logical reason for doing them. Mickey, on the other hand, was able to reasonably interpret the situation. However, he got hung up with his answer not matching his impression of how many popcorn kernels there were—it looked like more than 5824 kernels in the container.

"Teaching is difficult, but interesting and challenging," Mr. Thomas thought. "This experience really reinforces for me how important it is to carefully watch and observe my students as they work. This is critical if I am going to be able to support each student's learning."

Activity: Faces in a Crowd

There are some circumstances when it is impossible to get an exact count; only an estimate is possible. Using a picture of a crowded street or square, ask students to estimate how many people there are. Working in groups, students should decide on a method of estimating the size of the crowd, and list factors they considered in arriving at their estimate. Sharing the groups' results will reveal the problems and benefits of making estimates.

Activity: How Many Pages to the Centimeter?

How Thick is One Page?

Ask students to select several thick books from the classroom or the library. Without first looking, estimate the number of pages in each book. Then students should measure the thickness of each book using a centimeter ruler and record the measure along with the number of pages. Ask them to calculate the number of pages, in a 1 centimeter section and in a 1 millimeter section, and then check to see if their calculations match results when they actually measure and look at page numbers. Do students get the same results if they repeat the exercise? What happens if different students measure the same book? Students should compare their results for different books. How different were their findings for number of pages to the centimeter? Could they be sure that their estimate would be accurate? What factors could cause errors?

Students should compute the thickness of a single page of a book. This provides opportunities for students to work with decimal numbers that are very small.

Assessment Activity: Areas of Circles

There are many ways a teacher can assess students' understanding of various mathematical concepts. The following examples illustrate how the same problem can be asked in various, and in increasingly complex, ways. Even the simplest question (#1) is more complex than typical textbook exercise that asks students to simply compute areas of various circles. (Middle school teachers know that many students have difficulty even doing this.)

The problems below are intended for all students. It is assumed, however, that success for all students will require a nontraditional classroom. Not all students will have the chance to be successful, unless we change the climate of the classroom to include expectations that students are to think and reason, to explain and justify their thinking, make connections between ideas within as well as outside of mathematics, while frequently interacting with the teacher and other students.

Assessment Questions	Comment
1. How many square inches larger is a circle with a diameter of 12 inches than a circle with a diameter of 6 inches?	Even though examples 1 and 2 are quite traditional—there is no context and each requires a specific numerical answer—they ask for more complex thinking than asking students to compute the area of various circles. The first question is simply an exercise in apply the area formula and subtracting.
2. What fraction of a 12 inch circle is a 6 inch circle?	The second question has more possibilities for how students might approach and solve it. Some students might do the computation and be surprised when the ratio comes out to be .25. Others might see that the only difference in the area equations for the two circles is the value for r^2: 9 and 36. Others might see connections between areas of circles and other geometric shapes when a critical dimension is doubled.
3. If a 6-inch pizza sells for $2.75, what is the price for a 12-inch pizza? (Assume the price is proportional.)	This problem is put in a context, and requires an additional step, finding a price that is proportional. There is still one correct answer. The only way for the teacher to judge a student's understanding is by examining their written computation, unless the teacher asks a student to explain what he or she has done.
4. The eighth grade class is going to sell pizzas to make money for graduation events. It is going to sell two sizes: 6-inch (diameter) individual pizzas, and 12-inch group pizzas. What would be a fair price to sell the 12-inch pizza if the 6-inch pizza will sell for $2.75? Explain your reasoning.	This problem is more open than #3 and requires students to write about their thinking. There may be more than one correct answer, for example, if students think that it requires more work and time to make four 6-inch pizzas than one 12-inch, and so the price of the 12-inch pizza should be lower than the four 6-inch ones. All four fundamental standards are addressed here. The problem in this form requires reasoning and problem solving, making connections in a real situation. In explaining their reasoning, students are also asked to communicate mathematically.
5. The eighth grade class is going to make and sell pizzas to make money for graduation events. It is going to sell two sizes: 6-inch (diameter) individual pizzas, and 12-inch group pizzas. They have found the following costs for the ingredients: Premixed dough: $1.89 (makes four 6-inch pizzas) Pizza sauce: $2.29/bottle (enough for two 12-inch pizzas) Grated cheese: $1.59/pkg. (for one 6-inch pizza) What would be a fair charge for a 6-inch and for a 12-inch pizza? Explain your reasoning.	This problem is more complex and open than question #4. This is the first problem that requires that the students take into account the factor that the class wants to make money, so a profit has to be included. There could be many correct answers, depending on how students approached, solved, and explained their reasoning and their solution. While this problem is more like a real situations than earlier problems, it is still somewhat contrived. It is not a real situation for the students. It assumes that there will only be one kind of pizza, so students do not consider what they will sell. All prices are provided so students do not need to do research.

Cluster II: Patterns, Relations, and Functions

> *Mathematics is an exploratory science that seeks to understand every kind of pattern—patterns that occur in nature, patterns invented by the human mind, and even patterns created by other patterns. To grow mathematically, children must be exposed to a rich variety of patterns appropriateto their own lives through which they can see variety, regularity, and interconnections.*
>
> *Lyn Arthur Steen, On the Shoulders of Giants, p.8*

Pattern is the basis of mathematics. Patterns help people interact with and make sense of the world. Patterns in nature, such as snowflakes or the spiral of a pinecone are fascinating; many can be described mathematically using geometry, number sequences, and functions. Students can see patterns as they watch plants and animals grow; Fibonacci numbers were discovered by modeling a theoretical birth pattern of rabbits. Many mathematical patterns, such as odd and even numbers, square numbers, triangular numbers, and Pascal's triangle can be investigated both numerically and geometrically. Patterns are also central to literature, art, and music. Many of these too may be investigated mathematically.

Patterns, relations, and functions should be part of the mathematics curriculum from the time children start school. Young children can identify and extend repeating rhythmical patterns, verbal patterns, and visual patterns. They can see patterns and relationships among various objects by sorting and classifying them, identifying similarities and differences. They can find numerical patterns on a 100's chart and when they count by twos or threes. They can begin to develop an understanding of functions (and multiples) by making a list, for examples of numbers of insects and legs—one insect has 6 legs, 2 insects have 12 legs, etc.

As students mature, the work they do with patterns and functions will become increasingly mathematically sophisticated. While students will still identify patterns and relationships in a variety of real-world situations, they will identify and represent numerical relationships in tables and graphically. For example, students may show the relationship between speed and distance by making a linear graph showing the distance covered by automobiles going a constant speed. They may also collect time and distance data of a real trip (say, from school to downtown) that shows that an average speed only represents one aspect of the trip. This real data can be displayed on different, non-linear, graphs, that perhaps show actual distance versus time or one that shows speed versus time (the graphs drops to zero while at a stop light).

Much of the students' work with patterns and functions is a forerunner to the study of algebra. Generalizing from patterns and describing relationships using a variety of methods facilitates students' transitions to algebra. Algebra provides a mathematical structure for working with patterns. As indicated in *Algebra for the Twenty-first Century* (Proceedings of the August 1992 Conference, NCTM, p. 6), algebra can be broadly thought of "as a way to represent information involving both a process of reasoning and a structure that allow the representation to be used to gain a better understanding of the information. That is, a situation is modeled algebraically, manipulated according to the properties and rules of algebra, and the results are returned to the situation for interpretation.... This broad view of algebra allows a more unifying view of the 9-12 *Curriculum and Evaluations Standards* —algebra, functions, trigonometry, calculus concepts and analytical geometry—as a study of functions and structure."

Patterns and functions are important to such mathematical topics as transformational geometry, algebra, discrete math, trigonometry and calculus. Older students need to explore ideas about continuity, discontinuity, maximum and minimum. Real world optimization problems, such as those related to maximum profit earned or minimum height achieved, provide an opportunity to see the relevance of the concepts.

Math concepts formerly taught only in basic algebra courses are increasingly part of the culture and vocabulary of modern life. For example, the concept of positive and negative numbers is found in widely-used medical terminology such as "HIV-positive," or RH-negative." The concept of "exponential growth" appears in descriptions of medical or business phenomena such as the increase in the number of AIDS cases in the last ten years or the expected growth of the U.S. debt.

Life experience has afforded adult basic education learners with a broad base of real-world ties which can be readily linked to the concepts of equation, function, variable, and graph. From baby

formulas to chemical formulas, algebra offers a succinct way to define real-world situations that can aid adults in the home and in the workplace. ABE teacher Esther Leonelli offers two examples:

1. A person following a popular diet plan uses color-coded cards to represent the allowable daily food exchanges. These can be connected to the "formula" for a balanced daily caloric intake:

$$3F + 2V + 2M + 3B + 3f + 8P = \text{Daily Total}$$

where F stands for fruit, V for vegetable, M for milk, B for bread, f for fat and P for protein units.

2. A nursing home aide who plans on entering nursing training after earning his GED can learn to apply the algebraic proportionality rules used in figuring dosages and medications.

The opportunity to study algebra should be available to any adult basic education learner who may have missed it due to past educational experiences. Students whose academic background did not include algebra, or for whom the study was unproductive when it occurred, marvel at themselves when they, as adults, finally "get" what algebra is all about. As a matter of equity, algebra instruction should be made accessible to all adult learners, whether they were previously denied access by public school "tracking," or currently by adult education models of instruction which preclude the study of algebra as a "life skill."

NCTM Patterns, Relations, and Functions Standards

K-4 Standards

Standard 13: Patterns and Relationships
In grades K-4 the mathematics curriculum should include the study of patterns and relationships so that students can—

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

5-8 Standards

Standard 8: Patterns and Functions
In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can—

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems.

Standard 9: Algebra
In grades 5-8, the mathematics curriculum should include exploration of algebraic concepts and processes so that students can—

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems.

ABE Standard 7: Patterns, Relationships, and Functions
In the adult basic education classroom, curriculum design must include approaches to teaching about patterns, relationships, and functions which allow the learner to:

- explore, recognize, analyze, and extend patterns in mathematical and real-world situations;
- articulate and represent number and data relationships using words, tables, graphs, and rules.
- discover and use patterns and functions to represent and solve problems.

ABE Standard 8: Algebra
In the adult basic education classroom, curriculum design must include
approaches to teaching algebra which allow the learner to:

- represent arithmetic patterns and real-world situations using tables, graphs, verbal rules, equations, and explore the interrelationships of these presentations;
- understand the concepts of and recognize the use of variables, expressions and equations;
- develop confidence in solving equations in one or two variables using concrete, informal, and formal methods;
- apply algebraic methods to solve and represent a variety of test-related, work-specific or real-world mathematical problems.

NCTM Patterns, Relations, and Functions Standards

9-10 Standards

Standard 5: Algebra
In grades 9-10, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can—

- represent situations that involve variable quantities with expressions, equations, and inequalities;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and solve equations and inequalities;
- appreciated the power of mathematical abstraction and symbolism.

Standard 6: Functions
In grades 9-10, the mathematics curriculum should include the continued study of functions so that all students can—

- model real-world phenomena with a variety of linear functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions.

Standard 9: Trigonometry
In grades 9-10 the mathematics curriculum should include the study of trigonometry so that all students can—

- apply trigonometry to problem situations involving right triangles.

11-12 Standards

Standard 5: Algebra
In grades 11-12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can—

- represent situations that involve variable quantities with matrices;
- use tables and graphs as tools to interpret higher order equations, inequalities, and matrices;
- operate on matrices, and solve equations and inequalities;
- appreciated the power of mathematical abstraction and symbolism;

and so that, in addition, college-intending students can—

- use matrices to solve linear systems;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

Standard 6: Functions
In grades 11-12, the mathematics curriculum should include the continued study of functions so that all students can—

- model real-world phenomena with a variety of nonlinear functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;

and so that, in addition, college-intending students can—

- understand operations on, and the general properties and behavior of, classes of functions.

Standard 9: Trigonometry
In grades 11-12 the mathematics curriculum should include the study of trigonometry so that all students can—

- apply trigonometry to problem situations involving triangles;
- explore periodic real-world phenomena using the sine and cosine functions;
- apply general graphing techniques to trigonometric functions;

and so that, in addition, college-intending students can—

- understand the connections between trigonometric and circular functions;
- use circular functions to model periodic real-world phenomena;
- solve trigonometric equations and verify trigonometric identities;
- understand the connections between trigonometric functions and polar coordinates, complex numbers, and series.

Standard 13: Conceptual Underpinnings of Calculus
In grades 11-12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can—

- investigate limiting processes by examining infinite sequences and series.
- determine maximum and minimum points of a graph and interpret the results in problem situations;
- investigate limiting processes by examining areas under curves;

and so that, in addition, college-intending students can—

- understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
- analyze the graphs of polynomial, rational, radical, and transcendental functions.

Grades K-4: Patterns, Relations, and Functions Activities

	Activity	Comment
Activity: Kinesthetic Patterns	Provide ample experiences for young children to participate in kinesthetic patterns. (i.e. clap, clap, hop) Include music patterns and rhymes. Have students create visual patterns using common materials, such as shells or leaves.	Various multi-sensory experiences allow children to internalize language and mathematical concepts.
Activity: Attribute Block Trains	Have groups of four students use a set of attribute blocks to make linear pattern trains that differ by only one attribute. Students take turns placing one block at a time while naming and describing the shape (properties) and identifying the attribute that makes it different from the previous block. **Extension:** Make linear pattern trains of blocks with more than one attribute, or sort the blocks with Venn Diagrams. **Technology Connection:** *Gertrude's Secrets* or *Gertrude's Puzzles.*	As students work with attributes classifications, and patterns, they learn to reason mathematically; and to analyze and make generalizations about the characteristics of the attributes. Learning collaboratively helps students communicate more effectively and allows them the opportunity to explain and support their thinking and reasoning. Using computer software that extends the students' work with the blocks helps the students make connections and apply their knowledge as they prepare themselves for a highly technological world.
Activity: Patterns on a Hundred Chart	Children skip count on a hundred chart and color in the first 5 to 10 numbers in the sequence. Students look at the emerging pattern and predict the next numbers in the sequence. **Technology Connection:** Students use the constant function on a calculator to skip count. Students write the numbers in the pattern and write about the pattern they see, and describe the next numbers in the sequence.	These activities allow students to describe both spatial and numerical patterns. This helps reinforce number sense and prepares students for understanding the concept of multiplication.

Activity: Tessellating Pentominoes

Explore tessellations using different pentominoes. Have the students investigate, "Do all the pentominoes tessellate?" (A tessellation is a pattern that covers a plane without gaps or overlaps, such as floor tiling. A pentominoe is an arrangement of five equal squares where all squares have at least one side in common with another square.)

Open-ended investigations allow students to construct their own understanding of tilings.

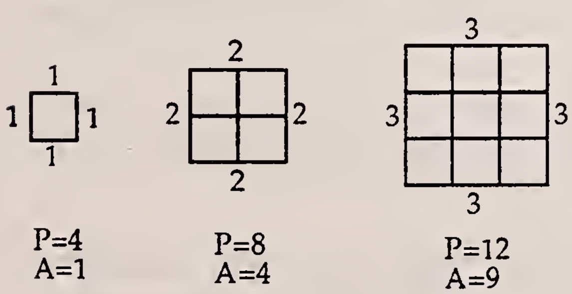
Activity: Square Functions

Using tiles, ask students to build larger and larger squares. Look at several different attributes of the squares, such as perimeter and area. Students begin to record their findings in drawings and tables.

Ask students questions such as:

- Describe the patterns you see. Can you continue the pattern?
- What would be the perimeter of a square with a side length of 9? What would be its area?
- What would be the side length of a square with a perimeter of 36? What would be the side length of a square with an area of 36?
- What would be the area of a square with a perimeter of 10? How could you show this?

Students in third and fourth grades can begin to explore functional relationships. Using tiles is a concrete way to begin to look at patterns and functions. This activity cuts across many of the Standards by integrating concepts from the Number, Geometry, and Measurement Standards as well as Patterns, Relations, and Functions.



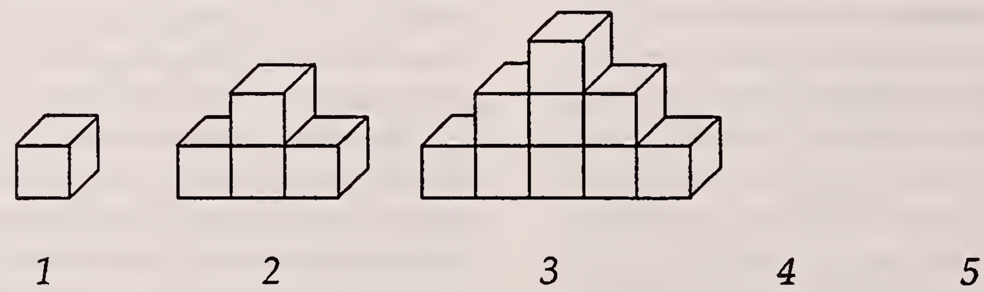
Length of side	Perimeter	Area
1	4	1
2	8	4
3	12	9
4	16	16

Grades 5-8: Patterns, Relations, and Functions Activities

Activity: Functions from Sequences of Cube Patterns

Investigating functions based on sequences of cube patterns allows students to work with a variety of functions at different levels of difficulty and to use a variety of approaches learning about functions.

Using cubes ask students to build a sequence of "towers," "pyramids," or other patterns. (It helps to use cards to identify the order of the patterns in the sequence.) Start with patterns, such as the following, that are fairly easy to predict how they continue. As each sequence is built, ask students to describe the pattern they see with words, and to predict and build the next two shapes.



Ask students to investigate questions such as the following.

If we continued building this sequence pattern what would be:

- the number of cubes in the 7th shape? the 10th shape? the 100th shape?
- the height of the tallest tower in the 7th shape? the 10th shape? the 100th shape?
- the number of cubes in the top layer of the 7th shape? the 10th shape? the 100th shape?
- the number of cubes in the 7th shape (10th shape, 100th shape) which would have exactly two faces (three faces, four faces) painted if the outside of the entire shape was painted?

Ask students to organize the information they've gathered in several different ways. For example, by making a table, describing the pattern in writing, writing an equation, and plotting points on a coordinate graph.

Ask students to invent new cube pattern sequences and investigate some questions about them.

Activity: Guess the Rule

Play guess the rule with the class. Secretly choose a rule, such as doubling a number and adding 5. Tell the students to give you an input number (x), you will apply your rule, and tell them the resulting, or output, number (y). Record the input and output numbers in a table in the order they are asked. When some students figure out what you're doing to each number, they are not to state the rule out loud because others should have the opportunity to figure out the rule. Those students can become the experts providing the output number.

When most of the class has figured out the rule, students should state it in words and in equations. Encourage students to share different ways of stating the rule and writing the equation,
such as $y = x + x + 5$ and $y = 2x + 5$.

Play the game frequently, with different students selecting the rule.

Introduce graphing of equations on the coordinate grid by asking students to plot ordered pairs of numbers from a "Guess the Rule" table. Look for the patterns different rules make.

Play Guess the Rule using the coordinate grid by investigating changes made to graphs of functions.

For example, What would the equation be if:

- the line were moved up (or down) but kept parallel to the original line?
- the line were less (or more) steep?
- if the line was flipped or reflected along the y axis? along the x axis?

Students can draw the new graph, and then find the ordered pairs from the graph to play a new Guess the Rule.

Grades 9-12: Patterns, Relations, and Functions Activities

Activity: "Ask Marilyn"

The following problem appeared in the "Ask Marilyn" column of the November 7, 1993 Parade magazine.

> Half the employees in a firm went to lunch at noon. Since then, 25 have returned and 7 others have gone out. At this point, there are twice as many people working as there are people out to lunch. How many people are employed by the firm?

Have students work on this problem alone for a few minutes, then have them form groups to compare their solutions and pool them to come up with an agreed upon group solution to the problem. Ask each group to write an explanation of their solution to the problem.

If students have not already done so, ask them to express the information in the problem in the form of an algebraic equation, For example, students may write:

$$\frac{1}{2}E + 25 - 7 = 2\left(\frac{1}{2}E - 25 + 7\right)$$

or, perhaps:

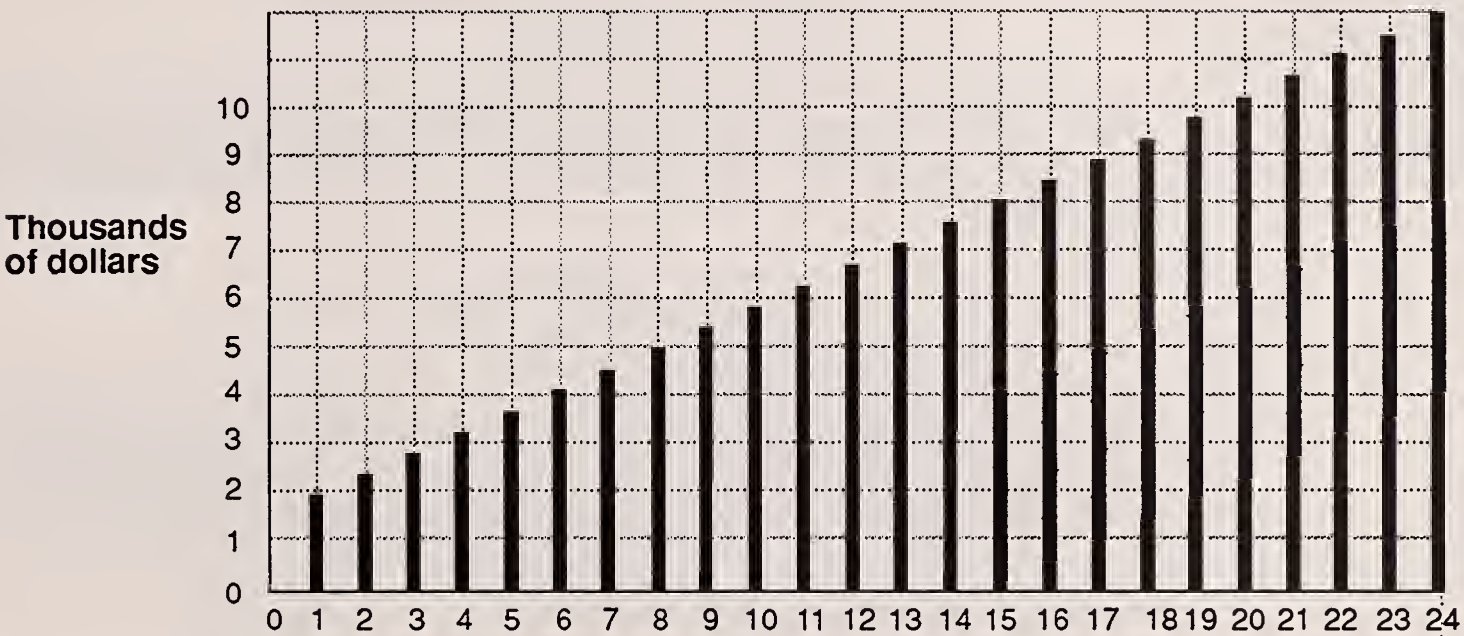
$$\left(\frac{N}{2} - 25 + 7\right) \times 2 = N - \left(\frac{N}{2} - 25 + 7\right)$$

If different groups of students come up with different equations, it provides an opportunity for them to try to understand how another group arrived at their equation, and to see that there is more than one correct way to represent a problem symbolically.

Ask each group of students make up a similar problem. Then ask them to share and try to solve each other's problems. Explain to them that the problem above was contributed to "Ask Marilyn" by Bill Saltzman of St. Louis, Missouri. Ask them to write up their problems and send them to the address listed at the bottom of the "Ask Marilyn" column.

Activity: Comparing Car Loan and Car Leasing Payments

Have students bring in automobile advertisements from several newspapers and select cars they would like to buy. After they have determined the price of the cars they would like to buy, have them contact local car dealers and/ or banks to find out the down payments and monthly payments for each car selected, if purchased over a two-year period. Have them graph the cumulative payments, using graph paper, scientific calculators, graphing calculators, or spreadsheets on computers. For example, they might graph:



They might also figure out the amount that will go toward paying off the principal and the total amount of interest paid.

Have students make similar graphs showing the payments for leasing the cars for the same period of time, with an option to buy at the end of the period. Have them graph the loan payments and lease payments for the same car on the same graph. What do they notice?

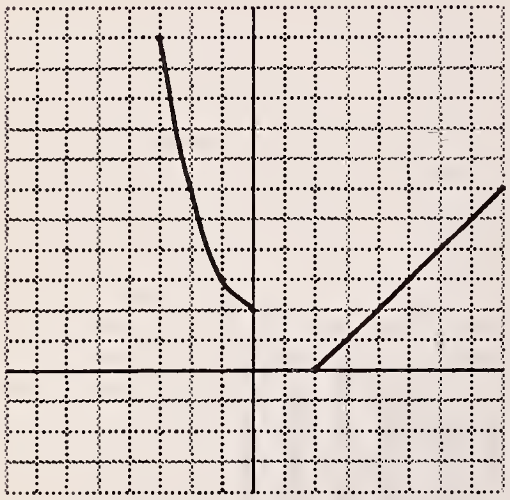
Ask the students to compare the two methods of purchasing a car and identify some factors which might influence a buyer's decision: amount of initial payment, trade-in value, total cost, anticipated cost of repairs, option to sell at the end of two years, and size of final payment. Have them write a paragraph comparing the two methods or arguing for the method they think is best.

This activity might be extended by exploring the formula used by the bank to calculate monthly payments. The formula for the monthly payments on a loan of amount A at an annual rate of interest i over a period of n years is:

$$\textit{Monthly payment} = A \times \left[\frac{\frac{i}{12}}{1 - \left(1 + \frac{i}{12}\right)^{-12n}} \right]$$

Activity: Exploring Continuity

Give the students the following graph:



Ask them to sketch in two graphs for the interval $0 < x < 2$, one which makes the graph continuous, one which makes it discontinuous. Ask the students to share and discuss some of their sketches, to establish that there are many different possibilities.

Present the definition of a continuous function:

$$\lim_{x \to c} f(x) = f(c) \text{ for all } c.$$

Then ask them to write formulas for the function, choosing a formula for the interval $0 < x < 2$ which makes the function continuous. Students might write, for example:

$$f(x) = x^2 + 2 \text{ for } x \leq 0$$
$$f(x) = ax + b \text{ for } 0<x<2$$
$$f(x) = x-2 \text{ for } x \geq 2$$

Other students might be challenged to find a quadratic formula for the interval $0 < x < 2$, or to write a formula for that interval which makes the function discontinuous at exactly one point, at 0, at 2, or at 1.

Activity: Applying Recursive Processes

Algebra for the Twenty-first Century: Proceedings of the August 1992 Conference presents the following problem:

> A patient is taking approximately 16 ml of medicine every four hours for a long period of time. How much of the medicine will eventually be in her blood? Assume the body eliminates 25% of the drug in every four hour interval. (p. 22)

This is a problem which students can investigate in several different ways. They might, for example, carry out a simulation, set up a table by using arithmetic calculations, use a spreadsheet, or use the formula for the sum of a geometric series. They can consider the importance of two levels associated with most medicines, the minimum level below which the drug does not have the intended positive effect, and the maximum level above which the drug is toxic. This and similar problems provide realistic examples of recursive functions. "Recursion is a very important process employed by many computer programs to deal with a variety of problems in many disciplines." (p. 25) (For a more complete description of this activity, see pages 22 - 25.)

Activity: Experimenting with Rebounding Balls

Cooperative Learning in Mathematics: A Handbook for Teachers, describes an activity which provides the opportunity to connect experimentation and prediction, averages, algebra, modeling, and geometric progressions. Begin by asking students to work in groups to drop a ball and observe how high it rebounds on the first bounce. Ask students to find the average first rebound from a given height and to predict the height of the second rebound. Ask them to test their predictions. Ask them to develop a model for the situation and predict the height after 4, 5, 25 falls. (For a more complete description of this activity, see pages 222-224.)

This activity might include a comparison of the elasticity of similar and different balls. How much variation is there among tennis balls from the same can? How much variation is there among different brands of tennis balls? How do tennis balls compare with basketballs and with dime-store "hard" balls?

Activity: Compound Interest Problem

Hand out a pamphlet from a bank which summarizes interest rates and methods of compounding on various types of accounts. Explain the difference between simple and compound interest. Have students investigate how money grows using compound interest by filling in a table using calculators or computers with spreadsheets.

Year	Amount in account at beginning of year	Interest earned	Total amount at year's end
1	\$100	\$100(.05)	100 + 100(.05) = \$105
2	\$105	\$105(.05)	105 + 105(.05) = \$110.25
3			
4			
5			

Ask the students to use graphing calculators or computers to graph the results from the table. Starting with \$100 at 5% interest, how long would it take for your money to double in value? Suppose you started with \$250?

Have the students experiment with other rates of interest, and then have them derive the formula for the accumulated value, A, given principle, P, and interest, i, at the end of n years:

$$A = P(1 + i)^n$$

Ask the students to return to the original problem and calculate the amount if the interest is compounded semiannually:

$$A_1 = \$100\left(1 + \frac{.05}{2}\right) = \$102.50$$

$$A = A_2 = \$102.50\left(1 + \frac{.05}{2}\right) = \$105.0625$$

Point out that "effective rate of interest" is a term often seen in bank literature and ask students what the "effective rate of interest" is for this example.

Ask the students to extend this by computing the effective rate of interest if the interest is compounded quarterly, monthly, and weekly. Ask them to predict the amount at the end of the year if the interest is compounded every second. Then ask them to test their predictions. Students should come to understand that there is not an infinite amount of money to be had by compounding the interest more often. Ask them to use graphing calculators or graphing software to make a bar graphs of the results.

Assist students to derive the formula for interest compounded k times per year:

$$A = P\left(1 + \frac{i}{k}\right)^k$$

Use the charts, graphs, and formula to introduce and discuss the concept of a limit. In this case:

$$\lim_{k \to \infty} A = P\left(1 + \frac{i}{k}\right)^k = P\,(e)^i$$

Cluster III: Geometry and Measurement

Before children begin school, they already have developed a knowledge of and a curiosity about the physical and spatial world around them. The teacher has the opportunity and the responsibility to use this natural curiosity to build a mathematical foundation in geometry and measurement. The study of these areas allows for practical applications of mathematics and for the development of many skills and concepts. Students need geometry and measurement experiences that are presented in problem-solving situations and incorporated into number, patterns, and other mathematical ideas in order to increase their understanding of mathematics in real world contexts.

Students need to experience geometry and measurement by working with everyday objects and tools, as well as other concrete materials. In the early grades, the informal and intuitive studies of geometry and measurement are very important. For example, beginning measuring experiences with nonstandard units of measure (i.e., shoe, hand), spatial sense, size, and shape make young children's mathematical experiences personal and fun, and lays the foundation for understanding the need for standard units of measurement. As students explore with manipulatives such as blocks, geoboards, balance scales, measuring tools, to name a few, they develop a personal, physical understanding of mathematical ideas. Connections within mathematics, besides connections to other curriculum areas, practical situations, and real life should be explored, to enable students to apply what they learned, in subsequent learning situations.

Continuing the study of geometry and measurement in the middle grades should allow students to see the dynamic role that mathematics plays in the environment. As students use two and three dimensional models to investigate patterns and to develop spatial skills, they will see the practical uses of mathematics and the way mathematics is used in a wide variety of careers. In the middle schools, placing more emphasis on measurement and geometry can often interest students who have been turned off to or not successful in mathematics. Investigating problems that involve geometry and measurement can broaden all students understandings of mathematics and engage them with important mathematical ideas.

As the study of measurement and geometry becomes more formal in high school, students become better equipped for mathematical argumentation. Instead of emphasizing only two-column proofs, students should have opportunities to justifying their own conclu-

sions with less formal, but nonetheless, convincing arguments. It is important that the students' spatial reasoning and visualization skills be enhanced. Because of the advances in computer technology, the study of geometry has taken on a new focus relative to the 21st century.

As the study of measurement continues in the upper grades, students will need to be able to determine when an estimate or actual measurement is appropriate, choose suitable units and tools of measure, allow for errors in measurements, and design ways to assign numerical values to quantities. "Measuring diverse quantities makes connections within mathematics, especially to statistics, and outside, to the natural and social sciences" (*California Mathematics Framework*, page 85).

In the ABE classroom, teachers should use concrete activities (with nonstandard and standard units) to help ABE learners develop an understanding of the many measurable attributes of physical objects (length, time, temperature, capacity, weight, mass, area, volume, and angle). This is a natural way of building a vocabulary for measurement, and for comprehension of what it means to measure.

For ESL Learners, teaching measurement is very important as a cross-cultural component of mathematics and second-language learning, since many of these learners have used the metric measurement system more than the customary U.S. system.

NCTM Geometry and Measurement Standards

K-4 Standards

Standard 9: Geometry and Spatial Sense
In grades K-4 the mathematics curriculum should include two- and three-dimensional geometry so that students can—

- describe, model, draw, and classify shapes;
- investigate and predict the results of combining, subdividing, and changing shapes;
- develop spatial sense;
- relate geometric ideas to number and measurement ideas;
- recognize and appreciate geometry in their world.

Standard 10: Measurement
In grades K-4 the mathematics curriculum should include measurement so that students can—

- understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;
- develop the process of measuring and concepts related to units of measurement;
- make and use estimates of measurement;
- make and use measurements in problem and everyday situations.

5-8 Standards

Standard 12: Geometry
In grades 5-8, the mathematics curriculum should include the study of the geometry on one, two, and three dimensions in a variety of situations so that students can—

- identify, describe, compare, and classify geometric figures;
- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop an appreciation of geometry as a means of describing the physical world.

Standard 13: Measurement
In grades 5-8, the mathematics curriculum should include extensive concrete experiences using measurement so that students can—

- extend their understanding of the process of measurement;
- estimate, make, and use measurements to describe and compare phenomena;
- select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- understand the structure and use of systems of measurement;
- extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass;
- develop the concepts of rates and other derived and indirect measurements;
- develop formulas and procedures for determining measures to solve problems.

ABE Standard 10: Geometry and Spatial Sense
In the adult basic education classroom, curriculum design must include approaches to teaching geometry and spatial sense which allow the learner to:

- use geometry as a means of describing the physical world relative to all the arts and sciences;
- understand and apply geometric properties and relationships to concrete situations;
- visualize and represent geometric figures with special attention to developing spatial sense;
- identify, describe, compare, and classify geometric figures;
- relate geometric ideas to number and measurement ideas, including the concepts of perimeter, area, volume, angle measure, capacity, weight, and mass;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- apply the use of appropriate technologies to the study of geometry and spatial sense.

ABE Standard 11: Measurement
In the adult basic education classroom, curriculum design must include concrete and experiential approaches to teaching measurements which allow the learner to:

- understand the process and concepts of measurement;
- make and use exact and estimated measurements to describe and compare phenomena;
- select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- understand the structure and use of different systems of measurement.

NCTM Geometry and Measurement Standards

9-10 Standards

Standard 7: Geometry from a Synthetic Perspective

In grades 9-10, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can—

- interpret and draw three-dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions.

Standard 8: Geometry from an Algebraic Perspective

In grades 9-10, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can—

- translate between synthetic and coordinate representations;
- deduce properties of figures using transformations and using coordinates;
- identify congruent and similar figures using transformations;
- apply transformations and coordinates in problem solving.

11-12 Standards

Standard 7: Geometry from a Synthetic Perspective

In grades 11-12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can—

- deduce properties of, and relationships between, figures from given assumptions;

so that, in addition, college-intending students can—

- develop an understanding of an axiomatic system through investigating and comparing various geometries.

Standard 8: Geometry from an Algebraic Perspective

In grades 11-12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can—

- analyze properties of Euclidean transformations and relate translations to vectors;

so that, in addition, college-intending students can—

- deduce properties of figures using vectors;
- apply vectors in problem solving.

Grades K-4: Geometry and Measurement Activities

Activity: Floor Shapes

Outline a shape on the floor with masking tape. Have the students walk around the figure. Find objects in the classroom that are the same shape. Work in pairs to compare the sizes of the objects.

Extension: Have pairs of students construct a shape (i.e., a square) on the geoboard. Have students arrange the shapes in order of size from smallest to largest.

Literature Connection: *Shapes, Shapes, Shapes* by Tana Hoban or *The Shapes Game* by Paul Rogers.

Measurement Extension: How many child's shoe length long is each side of the figure? The distance around the figure?

Students need to be involved in geometric explorations of the world around them. Early experiences with shape, size, and position will better prepare students for the learning of other mathematical topics. Working in pairs allows for communication of mathematical ideas. Use of manipulatives such as geoboards provides a concrete understanding of geometric concepts.

Beginning experiences with non-standard units of measure (i.e. shoe), size, and shape lay the foundation for future learnings, and make young children's first mathematical experiences both successful and fun.

Activity: Ordering by Weight

Ask students to order three or four different objects by weight. First estimate the order by how heavy each object feels in their hands. Students then use a balance to directly compare the weights of the objects. They then order the objects from lightest to heaviest and explain their strategy.

Early experiences with the attributes of weight and size help students develop some of the process skills necessary for later mathematics and science learnings (i.e., comparing, observing).

Activity: Transforming Shapes

Ask students to cut a 3 cm by 4 cm rectangle in half on the diagonal. Describe the two new shapes. Are they congruent? How many different shapes can be made if the two triangles are pieced together so that one pair of congruent sides is completely shared? Describe the new shapes with a partner. Write about them. Be sure to include their properties.

Extension: Which new figure has the largest perimeter? the smallest? What can be said about the new shapes' areas? What new shapes have lines of symmetry?

Students need experiences that place geometry in a problem-solving situation. Combining shapes, partitioning shapes, and exploring the relationships between shapes help students understand geometric concepts and develop spatial sense. Making connections within mathematics (i.e.. geometry with measurement) also results in deeper comprehension of skills and the ability to apply knowledge to other situations.

Activity: Measuring Plant Growth	Estimate, measure, and record data on amount of soil, water and light used. Measure the height of the plants. Graph and compare the data. Draw the different stages of plan growth. Predict growth rate, ultimate height, and optimum growing conditions based on data collected.	Mathematics is an important piece of the science curriculum. Students need experiences making and using estimates of measures as well as actual measurements over a period of time in order to make predictions based on changing data. Making connections to other subject areas (i.e., science) and within mathematics (i.e., measurement with statistics) helps students to not only understand concepts more clearly, but to be able to make future applications of their learning.
Activity: Measuring Angles	Use a circle protractor to measure angles of different degrees. Make a model of each angle out of wax paper. Duplicate each angle on the computer screen using a LOGO program. Use the wax paper model against the computer-imaged angle to verify accuracy in using a protractor and in programming LOGO. Describe each angle in terms of number of degrees and type (i.e., acute, right, obtuse). **Extension:** Tear the corners off a variety of paper triangles to explore what happens when the three angles of a triangle are combined to form one angle.	Technologies and tools such as a protractor help students develop a greater understanding of concepts and prepare them for a world in which technology takes on an increasing role. The study of geometry is an essential knowledge base for many careers from architecture to wallpaper hanging. By allowing students to physically explore concepts, they become better able to construct their own meaning and apply it in future problem-solving situations.
Activity: Designing a "High" Chair	Manute Bol, Shaquille O'Neal, Shawn Bradley and many other NBA basketball players are over seven feet tall. Collect the heights of many adults to determine the average height. Since mass-produced chairs are based on the average adult height, design a chair for NBA centers. Students will need to use measurement techniques to determine proportions for length of leg, calf, torso, arm, etc.	Activities that present geometry and measurement in a real-world, problem-solving context lend themselves to the interest and success of all students. Different strategies and rationale for solutions will arise as students use ration and proportion, averaging, size, shape, height, length, and weight (etc.) to come to a conclusion. Students also need experiences with activities that lead them to determining whether an actual measure or an estimate is sufficient.

Activity: Wrapping Candy Cubes

A company wants to wrap their new candies with as little waste of paper as possible. The candies are shaped like one inch cubes. Have students work in groups to develop ways to cut wrappings out of 1 inch graph paper so that when the paper is folded into a cube, there are no overlaps and no paper waste. What is the greatest number of candy wrappers that can be cut our of an 8" X 11" piece of wrapping paper?

Extension: Would less paper be used if the company stacks the candies in cubes of four candies than individually? How about in cubes of 9 candies?

Students need hands-on, real world problem-solving situations in order to strengthen conceptual understanding of mathematical concepts and their connections to each other. Cooperative groups allow students the opportunity to develop and share multiple strategies and to support their own reasoning.

Activity: Marshmallow Models

Ask students to make 3-D geometric models using straws and miniature marshmallows or toothpicks and gumdrops. Look at the surroundings and the environment for objects that model a three-dimensional solid.

Students need to construct 2-D and 3-D figures with many different materials to be able to identify the properties and characteristics of figures and to be able to make connections to the real world.

Activity: Planning a Field Trip

Students determine times, distances, expenses, etc. for a field trip to Sturbridge Village. How much time is needed to move from place to place within the Village, time for lunch, gift shopping, etc. Determine how much time is needed to travel to and from the Village, and how much time could be spent there. Use a calculator to determine costs for travel, admission, and lunch. Is the trip feasible? Justify your reasoning.

Students need to be involved in open-ended activities that allow for the exploration of geometry and measurement concepts in a problem-solving context. The experiences need to incorporate these concepts into number, time, money, and other mathematical ideas as applicable. By justifying their own conclusions, students' reasoning, thinking, and communicating skills are enhanced. Calculators should be available, as well as other manipulatives, as a tool to solving the problem using any number of different strategies.

Activity: Mapping

Have students describe and create models of their homes and then prepare maps depicting their route from home to school. Include a scale model for distance.

Extension: Add coordinates to the map to facilitate communication of various locations.

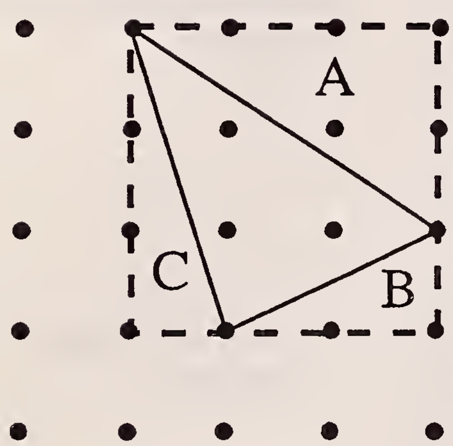
Students need to think about space and understand the connections between the representative model and reality. Concrete experiences in coordinates and mapping help develop their abilities to think abstractly and to read maps successfully. By including a distance scale, students have the opportunity to explore ratios and proportions in a scale context.

Grades 5-8: Geometry and Measurement Activities

Activity: Area on the Geoboard

Each student needs a 4 x 4 geoboard (25 nails), rubber bands, and dot paper. Ask students to make different triangles, find the area of each one, and record them on dot paper. What is the area of the largest triangle that can be made on the geoboard? the smallest? How many different triangles can be made with a given area. Ask students to prove how they know the areas of specific triangles, and to share proofs with the class using a transparency of the dot paper.

If students don't discover it on their own, you might share with the class the following method of proving the area of any figure that is made on a geoboard: Enclose the figure in a rectangle. The overall strategy is to find the areas of all pieces that are within the rectangle but outside of the figure and subtract their sum from the area of the enclosing rectangle.



Example:

Suppose the students has made this triangle on the geoboard (solid line), and then enclosed it in a rectangle (dotted line). The area of the enclosing rectangle is 9. There are 3 pieces (A, B, and C) within the rectangle but outside of the triangle.
Piece A is half of a rectangle that is 3 units long and 2 units high (6 square units), so the area of piece A is 3.
Piece B is half of a 2 x 1 rectangle or an area of 1 square unit.
Piece C is half of a 3 x 1 rectangle, or an area of $1\frac{1}{2}$ square units.
Subtracting the sum of areas of the three outside pieces
$(3 + 1 + 1\frac{1}{2}) = 5\frac{1}{2}$ from 9 leaves an area of $3\frac{1}{2}$ for the triangle.

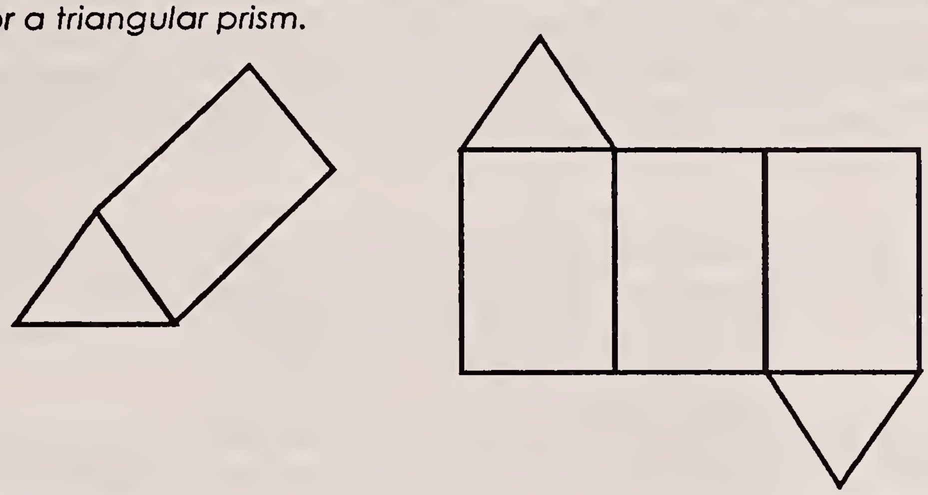
Ask students to find areas for all the different squares that can be made on the geoboard. Draw the squares on dot paper and explain how the area of each square was determined. (Besides the obvious squares with areas of 1, 4, 9, and 16, it is possible to make squares with areas of 2, 5, 8, and 10 on the geoboard. If the vertices of the squares don't have to be on nails, it is possible to make squares with other areas.)

Activity: Which Shapes Tessellate?

A tessellation is a covering or tiling of the plane with no gaps or overlaps. Students begin by investigating which of the Pattern Blocks shapes tessellate. Ask them to discuss why they think, for example, the trapezoid will tessellate, but the hexagon will not. Have students extend their investigation by seeing whether non-regular polygons will tessellate. Ask students to cut out of tag or construction paper multiple copies of a shape they have drawn and use them to investigate whether the shape will tessellate. Will all triangles tessellate? Will all quadrilaterals tessellate? What about pentagons? Ask students to come up with generalizations about which shapes will tessellate and why.

Activity: Exploring Polyhedra

Using wooden or plastic models of polyhedra, have students make different networks or patterns that will "wrap" a particular polyhedron. Students might "check" a pattern by cutting it out and folding it up. One example is shown below for a triangular prism.



Have students find the number of faces, vertices, and edges of different polyhedra, record their findings in a table, and find a rule or formula that shows the relationship among the numbers.

Technology Vignette

Seven new computers and software were placed in the 1,100 student middle school. Four department heads housed computers in their classrooms, and three computers were centrally located in the library/ media center. Computers were unused most of the day.

When Al and I decided to use the new geometry software in a geometry unit, we discovered the need for school policies that encourage integration of technology in classrooms. Although the administration and staff fully cooperated with our requests, it took precious time to make the arrangements to use two MACs in room 217 for three weeks. We shared our rationale with staff who were reluctant to "hand over" the computers, fearing they may never see them again, and we located furniture on which to set the MACs. We were ready! (Schools can promote meaningful use of technology in all classrooms by establishing a sign-out system that is teacher-friendly, by placing computers on carts, and by overseeing location and equitable use of the equipment.)

The 28 heterogeneously grouped seventh graders had not worked in cooperative learning groups in math, and technology was seldom used, but their behavior, attitudes, and involvement told us they wanted to do both. We designed four geometry tasks that each group was responsible for completing. One task was a tanagram exploration, another involved a traditional comparison of quadrilaterals, and another activity used the geometry software. Two of the seven small groups of 4 rotated onto the MACs each day, while the other 5 groups continued to work on the other tasks. In our planning, Al and I predicted that difficulty at the computer stations would monopolize our time (based on our experiences at teacher technology training sessions where adults freeze when confronted with an unfamiliar screen!). So I spent hours reading the software's documentation and working at a terminal trying to anticipate problems. I was haunted with the thoughts of groups getting lost or locked up somewhere in the software. Well, I wasted many hours worrying and studying because within 20 minutes of the first class, Al and I were learning about the geometry software from our students! Despite having never seen the software and equipped with a mere 4 page introduction to the software, 4 collaborating students navigated and problem solved with the program beyond my capabilities. Al and I became the learners in our classroom. I call this phenomenon "incredible intelligence" - an intelligence that is real but has no apparent source. I have a new respect for students' technological abilities, and I realize the need to design learning opportunities that tap and nurture these skills. I should not allow my understanding of technology to limit my students' understanding of technology. Using vaguely familiar technology in a classroom is not a big risk, since we are surrounded by potential experts, our students. (It was inspiring to observe students constructing their own understanding of the geometry software.)

P.S. Confident that we need not be graphing calculator experts (We were familiar with only half the keys!), Bev and I introduced graphing calculators to an 8th grade inclusion class.....with the same humbling yet inspiring results.

--Patty Forbes

Grades 9-12:
Geometry and Measurement Vignettes and Activities

Activity: Exploring the Angles of Polygons

Place students in groups of three and give them several large drawings of polygons: a quadrilateral, a pentagon, a hexagon, a septagon and an octagon. Ask them to determine the sum of the measures of the interior

Name of Polygon	# of sides	sum of angles
triangle	3	180
quadrilateral		
pentagon		
hexagon		
septagon		
octagon		

angles of each of the polygons, taking care to measure each angle carefully and trying to come to agreement within their group. Ask them to make a chart showing their results.

Name of Polygon	# of sides	sum of angles	# of triangles	sum of angles as a multiple of 180
triangle	3	180	1	180=1 x 180
quadrilateral				
pentagon				
hexagon				
septagon				
octagon				

Students then might be asked to look at the problem in another way, by drawing in all the diagonals that can be made from one vertex of the polygon. This would allow them to extend their chart.

Then ask students to predict the result for an n-gon (a polygon with n sides, where n can be any number) and to write a formula. At all stages they should be encouraged to discuss their ideas within their groups.

This activity might be extended to a study of tessellations. It might also be extended with a similar study of the sums of the measures of the exterior angles of polygons.

Teachers with access to computers and software for teaching geometry, such as the Geometric Supposer, Sketch Pad, or Logo, will want to use computers for these explorations.

Activity: A Geometric/Algebraic Study of Inverses

Ask the students to graph, separately, several functions (which have inverses) on grid paper. They might be asked, for example to graph:

$$f(x) = x^3$$

$$g(x) = x\sqrt[3]{x}$$

Then ask them to reflect each function in the line y = x, and sketch the result. Students may do this by using a mirror, or by using a special mirror for geometry (a MIRA, for example) which allows them to see, simultaneously, the reflection of the graph which is in front of the mirror and, through the mirror, the graph behind the mirror.

Ask the students to list some values of x and fill in a table by reading the y coordinates from the graph. Students should then try to derive the relationship between the x- and y- coordinates of the new graph. Ask them to write the new function and compare it with the original. What is the relationship between the two functions?

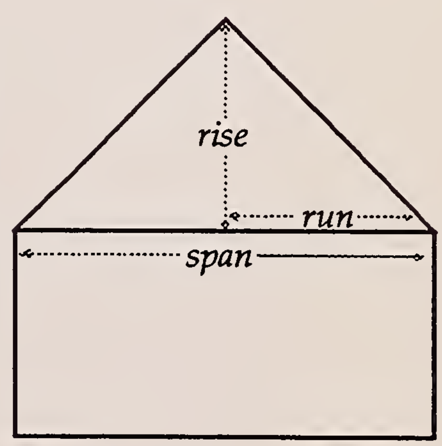
After students have done several pairs of functions, they should see that the inverse of a function can be found by reflecting about the line y = x.

Activity: Learning in the Building Trades

Provide the students with a sketch of a simple building and ask them to make a list of what they would need to know in order to figure out how much it would cost to build a wooden roof for the building. They may point out, for example, that they need to know:

- the length of the roof
- the length of the overhang
- the width of the roof
- the price of wood

Ask the class to choose some dimensions for the building, and then to gather any other needed information, such as the price of wood. They may not be aware that wood comes only in specific lengths. Ask the students to use the information they have gathered to prepare a cost analysis for this job.



As this activity is carried out, it will be useful to teach the students the vocabulary used in the construction industry. For example:

The pitch of a roof = $\frac{\text{rise}}{\text{span}}$

If $\frac{\text{rise}}{\text{run}} = \frac{4}{12'}$ *then pitch =* $\frac{4}{24}$ *or* $\frac{1}{6}$

Students might practice applying this vocabulary by doing examples such as:

If the pitch of a roof is $\frac{1}{6}$ *and the roof is 42 ft. wide, what is the rise?*

$$\frac{\text{rise}}{\text{span}} = \frac{1}{6} = \frac{x}{42}$$

The rise = 7 ft.

Vignette: Measurement and workplace needs

In workplace education and other adult education settings, instruction should focus on the measurement systems necessary to workplace and real-life needs. Teacher Judi Sulzbach describes her on-site class at a company where the metric system of measurement was becoming important.

"I wanted to see what my class knew about the word 'metrics', so we brainstormed a few ideas. I asked them to think of any word or phrase that came to mind when they thought of metrics. Some of their ideas were 'groceries, millimeters, grams, soda. measurement, charts and measuring tools. These showed ... that they were familiar with a lot more ideas on metric (measurement) than they realized."

"I gave each of them a ruler with cm-mm on one side and inches on the other. We set out measuring everything we could: chalk, erasers, pencils, books, paper clips, blackboard, wheel diameter, thickness and hole using all three of the measurements. We then looked at 2 liter soda bottles, soda cans with labeled "ml," and {quart and gallon] milk cartons. [This allowed them to] get an idea of how metric compares with our English measures...Later, we would do the same] with some of the measuring tools that they use on-the-job."

Cluster IV: Statistics and Probability

Knowledge of probability and statistics is becoming more and more important in everyday life. The ability to understand variability and uncertainty is necessary every time one reads the results of a Gallup poll or a report on the latest medical research findings. How accurate is an AIDS test? What are the chances that a nuclear power plant will have an accident? Is angioplasty as effective as coronary artery bypass surgery? In order to understand the arguments made by people on both sides and make informed decisions on questions like these, one has to have some understanding of the ideas of probability and statistical inference, and be aware of the uses and misuses of statistics.

It is becoming harder and harder to find an occupation or profession that does not require at least some knowledge of probability and statistics. For example, quality control techniques are beginning to be applied in service industries such as hospitals and banks; soon clerical and office workers will also need to have knowledge of probability and statistics. This is especially true in Massachusetts with its knowledge-intensive high-tech, financial services and budding biotechnology industries. Employment opportunities, even for students who do not go to college, will be limited without a knowledge of statistics.

People's intuitions about probability are often wrong. Therefore, it is critical for students to have many experiences with random phenomena from the earliest grades. The more opportunities students have to do hands-on activities with probability and data, the better base they will have on which to build new knowledge when they begin a more formal study of probability and statistics. Even in the later grades, topics should be introduced through activities and simulations followed by abstractions, rather than conversely.

The development of critical thinking in statistics, i.e. statistical thinking, not simply learning statistical procedures, should be emphasized. Students at all levels should be formulating appropriate questions; gathering and exploring data; organizing and describing data using graphs, charts, and tables; interpreting results; and developing a critical attitude toward the use of statistics.

Students need to investigate problems that use sampling techniques—problems which either can be answered only through statistics or can be answered better through statistics than through more exact methods. For example, to find out how many trees over six

inches in diameter are in a forest, the statistical approach to solving the problem is superior to trying to make an exact count. Trees in several sample areas of the forest can be counted and with the use of statistics the total number in the forest may be estimated. In some cases getting all possible information is not only cumbersome but also counterproductive. Crash-testing all cars, for example, would give us information about all, not just a sample of vehicles, but it would defeat the purpose of manufacturing them.

Concepts from probability and statistics should be introduced at the earliest grade levels and continued at with increasing depth and breadth as students develop intellectually. At grades K through 4 students generally will work with one variable at a time, and learn to make and read simple graphs. They will describe the data represented in such charts and graphs, and draw conclusions from them. K-4 students will learn that a sample can be representative of a population. In the middle school, students will begin to use scatter plots to explore the relationship between two variables, construct histograms for data which is grouped into intervals, and construct box and whisker plots. They also will learn more about sampling bias and randomness. Students in grades 9-12 will extend their knowledge to fitting nonlinear graphs to data. In 9-12, students will do a more detailed study of sampling methods and the role of sampling in judging the validity of statistical claims. This standard should not be viewed as advocating a separate statistics course at the high school level; rather the topics mentioned in it should be integrated with other mathematics courses.

Widespread use of personal computers has changed dramatically the way statistics is done. Today statisticians spend relatively more time trying to understand what their data has to tell them using these exploratory techniques, and relatively less time applying standard inferential techniques. What computers have done for statisticians, inexpensive statistical calculators and classroom software have done for students. They will rarely need to compute summary statistics with pencil and paper. Therefore, more emphasis should be put on interpretation of summary statistics—both student-generated and from daily life.

Adults are bombarded daily with results from statistical studies that can and do impact their lives. The adult learner is frequently aware that such numbers are continually used to define our existence, and generally displays a healthy interest in learning or re-learning the processes used for reaching such conclusions.

Adults know that decisions made on the basis of statistics may affect them daily, especially in discussions surrounding average scores required for tests, percentage of attendance at work or school, the price of transportation, the cost of child or health care, or even the number of immigrants allowed to become legal residents. It is imperative the adult basic education mathematics student understands not only how such statistical representations are often used, but also how such information may be misused.

NCTM Statistics and Probability Standards

K-4 Standards

Standard 11: Statistics and Probability

In grades K-4 the mathematics curriculum should include experiences with data analysis and probability so that students can—

- collect, organize, and describe data;
- construct, read, and interpret displays of data;
- formulate and solve problems that involve collecting and analyzing data;
- explore concepts of chance.

5-8 Standards

Standard 10: Statistics

In grades 5-8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can—

- systematically collect, organize, and describe data;
- construct, read, and interpret tables, charts, and graphs;
- make inferences and convincing arguments that are based on data analysis;
- evaluate arguments that are based on data analysis
- develop an appreciation for statistical methods as powerful means for decision making.

Standard 11: Probability

In grades 5-8, the mathematics curriculum should include explorations of probability in real-world situations so that students can—

- model situations by devising and carrying out experiments or simulations to determine probabilities;
- model situations by constructing a sample space to determine probabilities;
- appreciate the poser of using a probability model by comparing experimental results with mathematical expectations;
- make predictions that are based on experimental or theoretical probabilities;
- develop an appreciation for the pervasive use of probability in the real world.

ABE Standard 9: Statistics and Probability

In the adult basic education classroom, curriculum design must include approaches to teaching statistics and probability which allow the learner to:

- systematically collect, organize and describe data;
- construct, read and interpret tables, charts and graphs;
- make inferences and convincing arguments that are based on data analysis;
- evaluate arguments that are based on data analysis;
- develop an appreciation for statistical methods as a powerful means for decision making.

NCTM Statistics and Probability Standards

9-10 Standards

Standard 10: Statistics
In grades 9-10, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can—

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

Standard 11: Probability
In grades 9-10, the mathematics curriculum should include the continued study of probability so that all students can—

- use simulations to estimate probabilities.

11-12 Standards

Standard 10: Statistics
In grades 11-12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can—

- use curve fitting to predict from data:
- understand and apply measures of central tendency, variability, and correlation;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
- analyze the effects of data transformations on measures of central tendency and variability;

and so that, in addition, college-intending students can—

- transform data to aid in data interpretation and prediction;
- test hypotheses using appropriate statistics.

Standard 11: Probability
In grades 11-12, the mathematics curriculum should include the continued study of probability so that all students can—

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- understand the concept of a random variable;
- create and interpret discrete probability distributions;
- describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed;

and so that, in addition, college-intending students can—

- apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and chi square.

Grades K-4: Statistics and Probability Activities

Students need to have multiple experiences organizing and interpreting data in various forms, such as real graphs; picture, bar, and circle graphs; tables; and charts. Students' initial experiences with graphs should be with real graphs, in which real objects are placed on the graph. For example, if the students are creating a graph of the types of shoes worn by classmates, each student would place one shoe on a large floor graph. This information could then be translated into a bar graph, emphasizing one-to-one correspondence.

Statistics and probability have strong connections to topics in science (i.e.,. weather) and social studies (i.e., demographics).

Activity: Attendance Graphs

Start a daily "attendance" graph. Each day as they arrive at school students add their name to a graph for the question of the day. Examples of questions of the day include:

- What were you doing at 7:00 last night? (watching TV, doing home work, eating, playing, other)
- How many pets do you have?
- What is your birthday month?
- How many minutes did it take to get to school this morning?

Working with a partner, students take turns selecting a question of the day and preparing a graph for results. The question for the next day is presented prior to dismissal time so that students may collect any needed data.

Activity: Is the Game Fair?

Cut a small hole in the cover of an empty plastic container, large enough for dried lima beans to fall through, one at a time. Place six lima beans labeled 1-6 in the container, and replace the cover. Three to four players each choose a different sum—a number from 3 to 11. Players take turns, shaking the container, allowing two beans to fall out. A player scores one point if the sum of the numbers on the two beans matches the player's chosen number. Play continues until one player has scored 5 points.

Activity: What's in the Bags?

Give each cooperative group two paper bags filled as follows: Bag A– 5 red and 5 blue cubes or counters; Bag B– 3 red and 7 blue cubes or counters. Using the first Bag A, tell the students to take one cube from the bag, record the color on a bar graph, and return the cube to the bag. Do this 30 times. Then do the same procedures with Bag B and a second graph. Have students compare the two graphs and discuss the differences. Ask for reasons why the graphs are different. Display all the graphs for Bag A and all the graphs for Bag B together. Ask the students to notice patterns or trends and make predictions about the contents of the bags.

Grades 5-8: Statistics and Probability Activities

Activity: What is the Best Buy?

Working in groups, students identify a product and find the "best buy." The product should be something students are interested in such as jeans, sneakers, ice cream, or fast food. The entire class may decide to investigate the same product or each group may select its own product. If groups select different products, it is valuable to have at least two groups select the same product so that they can compare findings.

Students collect data about their product, for example, by visiting different stores and reviewing newspaper ads. As issues come up, students discuss them within their group, or possibly with the whole class. For example, students will need to grapple with what does "best" mean? Does it mean that the lowest price pair of jeans is the best buy regardless of its quality? or that the largest container of ice cream should be bought because the unit cost is lower even though I don't have a freezer to keep an uneaten portion?

As students narrow their investigations, as students to keep track of their decisions and why they were made. Ask students to distinguish between the kinds of data they are collecting as to whether they can be quantified.

Students prepare reports of their findings to present to their class. Report might include at least one graph of quantifiable data and one graph of categorical data.

Activity: Daily Lottery

So that students gain an understanding of how slim their chances of winning a lottery or contest are, conduct a daily lottery every school day throughout the year. Once the routine is established, it takes less than five minutes a day.

Each day as students enter the classroom, each student chooses a four-digit number for the day and records that number on a chart on the bulletin board. At the end of the period, the teacher selects a four-digit number by randomly drawing numbered tiles from a bag containing ten tiles numbered from 0-9. After each draw, the tile is returned to the bag before the next draw. A winning number has to have all four-digits correct and in the correct order.

The prize for having a winning number should be determined in advance by the teacher, or perhaps through a class discussion. It could be as simple as a recognition of the winner, or it might be replacing the teacher as the person who draws the four tiles each day, being the first to be excused at the end of the period, or being permitted to throw away the student's lowest test grade. Keep track of the number of winners and the number of days between winners. Periodically discuss with the class what they are learning about chance.

Grades 9-12: Statistics and Probability Activities

Activity: Fitting a Line to Data

Ask students to bring in two register tapes each from grocery stores. Have them plot the number of items and total cost on the tapes as ordered pairs on a rectangular coordinate system. Ask them to draw a straight line through the middle of the points on the graph by using a clear plastic ruler, a piece of uncooked spaghetti, or some other device that allows them to see the points on both sides of the line. Instruct them to adjust the device until the line goes through the center of the scatter of points and there are approximately the same number of points on each side of the line.

Next ask the students to find the equation of the line they drew by finding two points on the line whose coordinates can be read fairly accurately. Ask them to form small groups to compare their lines and to discuss reasons for the differences.

This activity can be extended by asking students to use a statistical technique such as the least squares regression or the median fit line to find the line that best fits the data, rather than fitting a line to the data by eye. Students could also be asked to use graphing calculators or computer software to fit the line to the data.

Depending on the students' level of performance, they could be asked to try fitting nonlinear functions, for example, quadratic, exponential, or logarithmic curves to sets of data, thus connecting statistics to other branches of mathematics.

Activity: Simulating a Population Distribution

Provide your students with the following information.

> In China, because of overpopulation, the government wants to limit the number of children born. Families are allowed, therefore, to have only one child. Many parents in China, however, as in many countries, want to have a son to take care of them in their old age. This is especially true in rural areas. As a result, many couples defy the government and keep having children until they have a son.

Ask the students to consider a possible change in the governmental policy. Suppose the Chinese government policy were to allow couples to keep having children until they had exactly one son. Ask the students to work in groups to compare the results of the two policies, assuming for now that couples will follow whichever policy is enacted. Ask the students first to discuss the questions listed below and record their predictions. Discussing their predictions and their reasoning with the class will help students develop their critical thinking and oral communication skills. Then ask the students to use random numbers and/or flip coins—for 100 couples—to simulate the results for each policy. Because each group of students will carry out an independent simulation, the results can be compared to see how similar or different they are. This will help students develop an understanding of random variation, a concept extremely important to both probability and inferential statistics.

- What is the average number of children per family?

- Are more boys born or more girls?

- What is the average number of girls per family?
- What is the average number of boys per family?
- Does the Chinese population increase, decrease, or remain the same?

As the students are working, help them recognize and articulate the assumptions they are making. Do they assume that the probability of the first child being a boy is .5? of being a girl is .5? Do they consider the possibility of multiple births? Do they assume that the sex of a second child (or third) is independent of the sex of the first child? Ask the students to discuss as a class whether the assumptions they are making are realistic or not, and, if not, why not.

Have students research the actual probability of male and female births. (One hundred six male babies are born for every 100 females, thus the probability of a boy is actually .515, and that of a girl .485.)

Have the students use random numbers to carry out the simulations using these more exact probabilities.

This activity offers possibilities for connecting mathematics and social science. Students might do research on topics related to world population growth, a topic which is periodically covered in the media. The simulations could be done in math class, with study of the social issues in a social science class.

Activity: Critiquing Statistics in the Media

Ask the students to bring in examples of statistics in the media. Ask them to analyze and critique the use of the statistics, and/or to write alternative uses of the statistics.

In analyzing statistics in advertisements, students might contrast the impression given by the advertisement with what the text actually says. For example, a recent advertisement included the statement, "More than 98% of all Chevy trucks sold in the last 10 years are still on the road." Students should understand that a quick reading might give the impression that 98% of the trucks sold 10 years ago are still on the road, or that all Chevy trucks last at least 10 years. With careful reading they should be able to determine that all of the trucks still on the road might have been sold in the last two or three years. Students should understand that the figure quoted may be accurate or inaccurate, but that even if it is reasonably accurate it is potentially misleading as used.

Students might also be asked to analyze and rewrite newspaper stories. For example, they might be given the following clipping which appeared on the front page of The Boston Globe.

North End residents opposed, survey says

> Opponents of a depressed Central Artery released a survey yesterday that they said showed most residents of the North End and areas surrounding it do not support the proposed project. The sampling showed that 61 percent of the persons interviewed are either opposed to the project or have no opinion.

After students have an opportunity to discuss the impression they get from the short introduction they could be given the complete article, which gives the following survey results:

opposed	29%
in favor	39%
undecided	18%
unfamiliar with project	14%

If the students have not already commented on it, point out to them that the opponents of the artery released the results of the survey. This can serve as a reminder that it is important to ask relevant questions before making judgments. Ask them to rewrite the short clip from the point of view of a supporter of the project, including a header—for example, "More than 70% percent of residents not opposed to Central Artery."

Point out to students that they will be exposed to statistics in the media and advertising throughout their lives, and that learning to recognize valid and misleading uses of statistics is an important and useful skill.

In March, Shelley brought in two graphs based on physical fitness. It examined the Healthy People 2000 Campaign which plans to reduce the number of Americans who are defined as medically "sedentary," no leisure time exercise. One related the national, state, and Year 2000 goal percentages. The other related percentages for the state's most sedentary: those 65 and older, blacks, those with low income, and Hispanic women.

The class exploded! Evelyn in particular was angered by this graph. She said, "They never asked me. Who did they ask?" Another student pronounced, "They are always putting black people down." Ethel said, "They don't know what black people do in their houses. If they know what's going on, why don't they help us?"

Appendix 1:
Recommendations for the Mathematical Preparation of Teachers of Mathematics

From: *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*, An MAA Report by the Mathematics Association of America, Committee on the Mathematical Education of Teachers, James R. C. Leitzel, Editor, 1991.

Standards Common to the Preparation of Mathematics Teachers at all Levels

Standard 1: Learning Mathematical Ideas

The mathematical preparation of teachers must enable them to:

- *become independent learners, capable of doing and learning mathematics on their own;*
- *develop their own processes, concepts, and techniques for solving problems;*
- *exercise mathematical reasoning through recognizing patterns, making and refining conjectures and definitions, and constructing logical arguments, both formal and heuristic, to justify results.*

Standard 2: Connecting Mathematical Ideas

The mathematical preparation of teachers must provide experiences in which they:

- *develop on understanding of the interrelationships within mathematics and an appreciation of its unity;*
- *explore the connections that exist between mathematics and other disciplines;*
- *apply mathematics learned in one context to the solutions of problems in other contexts.*

Standard 3: Communicating Mathematical Ideas

The mathematical preparation of teachers must enable them to:

- *develop skills in both written and oral communication of mathematical concepts and technical information;*
- *learn to communicate effectively at various levels of formality and with people who have differing levels of mathematical insight;*
- *understand and appreciate the power of mathematical language and symbolism in the development of mathematical concepts.*

Standards Common to the Preparation of Mathematics Teachers at all Levels (cont'd)

Standard 4: Building Mathematical Models

The mathematical preparation of teachers must include experiences that enable, motivate, and encourage them to analyze real-world situations through the use of whatever mathematical ideas or quantitative strategies are available. In particular, they should be able to:

- *work with a given model;*
- *recognize constraints inherent in a given model;*
- *construct models to analyze real-world settings and use symbols and reasoning in analysis;*
- *convert among representations (graphical, numerical, symbolic, verbal) that reflect quantitative constraints in a given real-world setting.*

Standard 5: Using Technology

The mathematical preparation of teachers must include experiences in which they use calculators and computers:

- *as tools to represent mathematical ideas and construct different representations of mathematical concepts;*
- *to engender a broad array of mathematical modes of thinking through use of powerful computing tools (including graphers, curve filters, and symbolic manipulators);*
- *to develop and use alternate strategies for solving problems.*

Standard 6: Developing Perspectives

The mathematical preparation of teachers must include experiences in which they:

- *explore the dynamic nature of mathematics and its increasingly significant role in social, cultural, and economic development;*
- *develop an appreciation of the contributions made by various cultures to the growth and development of mathematical ideas;*
- *investigate the contributions made by individuals, both female and male, and from a variety of culture, in the development of ancient, modern, and current mathematical topics;*
- *gain an understanding of the historical development of major school mathematics concepts.*

Recommendations for the Mathematical Preparation of Teachers of Mathematics

Standards for the Elementary Grades (K - 4)

Standard 1: Nature and Use of Number

The mathematical preparation for teachers of the elementary grades must provide experiences in which they:

- investigate the role of numbers as a logical, predictable system for expressing and relating quantities;
- analyze and compare features and basic computational techniques in selected numeration systems in use today and in the past;
- explore the operations, properties, and uses of numbers, fractions and decimals;
- use estimation and mental arithmetic, calculators, computers, paper and algorithms, and manipulative materials, in solving a wide variety of problems.

Standard 2: Geometry and Measurement

The mathematical preparation for teachers of the elementary grades must provide experience in which they:

- use a variety of tools, physical models, and appropriate technology to develop an understanding of geometric concepts and relationships and their use in describing the world in which we live;
- make and interpret measurements of many kinds of two and three-dimensional objects;
- formulate and solve problems whose solutions require spatial sense.

Standards for the Middle Grades (5 - 8)

Standard 1: Number Concepts and Relationships

The mathematical preparation of middle-grade mathematics teachers must include experiences in which they:

- develop a practical, concrete sense of number;
- use physical materials and models to explore fundamental properties of number systems;
- develop conjectures and intuitive proofs of number theoretic properties;
- investigate number sequences, patterns, and functional relationships;
- explore the measuring of infinity and its role in the study and historical development of topics such as geometry and calculus.

Standard 2: Geometry

The mathematical preparation of middle-grade mathematics teachers must include experiences in which they:

- investigate properties and relationships of shape, size, and symmetry in two- and three-dimensional space;
- explore concepts of motion in two- and three-dimensional space through the investigation of rotations, reflections, and translations;
- present written and oral arguments to justify conjectures and generalizations based on explorations;
- become familiar with the historical development of Euclidean and non-Euclidean geometries.

Standards for the Secondary Level (9 - 12)

Standard 1: Number Concepts and Properties

The mathematical preparation of teachers of grades 9-12 must include experiences in which they:

- explore and discuss the properties, relations, and extensions of the natural numbers, integers, rational, real and complex numbers;
- employ an understanding of number concepts and properties to investigate mathematical concepts and applications in diverse settings;
- study the historical development and significance of same major number-theoretic ideas and their applications.

Standard 2: Geometry

The mathematical preparation of teachers of grades 9-12 must include experiences in which they:

- model features of the world in which we live using different geometries;
- develop geometric concepts both synthetically and algebraically using coordinates and vectors;
- use geometry as a source of mathematical models for a variety of applications;
- employ geometric reasoning as a problem solving strategy;
- become familiar with the historical development of non-Euclidean geometries and the questions relating to the parallel postulate involved in this development.

Recommendations for the Mathematical Preparation of Teachers of Mathematics

Standards for the Elementary Grades (K - 4)

Standard 3: Patterns and Functions

The mathematical preparations for teachers of the elementary grades must provide experience in which they:

- recognize the study of patterns as an underlying, fundamental theme in mathematics;
- create and use pictures, charts, and graphs to recognize and describe mathematical relationships;
- develop the use of variables and other algebraic notation as an efficient and natural way to describe relationships.

Standard 4: Collecting, Representing, and Interpreting Data

The mathematical preparation for teachers of the elementary grades must provide experience in which they:

- collect and interpret data represented in different ways;
- conduct sampling experiments to develop an appreciation for randomness;
- explore empirical probability from data they have collected and relate it to theoretical probability based on a description of the underlying sample space;
- explore and compare various methods for representing data, both by hand and by using calculators and computers.

Standards for the Middle Grades (5 - 8)

Standard 3: Algebra and Algebraic Structures

The mathematical preparation of middle-grade mathematics teachers must include experience in which they:

- explore diverse examples of functions arising from a variety of problem situations and investigate the properties of these functions through appropriate technologies, including graphing utilities;
- use physical models, charts, graphs, equations, and inequalities to describe real-world relationships;
- explore and investigate properties of the integers, rational numbers, real and complex numbers (including order, denseness, and completeness);
- use concrete examples to explore selected algebraic structures such as groups, rings, fields, and vector spaces.

Standard 4: Probability and Statistics

The mathematical preparation of middle-grade mathematics teachers must include experiences in which they:

- collect data from experiments or surveys, organize and interpret data, and formulate convincing arguments based on appropriate data analyses;
- make inferences and informed decisions based on statistical methods;
- plan and conduct experiments and simulations to determine experimental probabilities;
- develop counting and other techniques useful in determining theoretical probabilities;
- identify the incorrect use of statistics by analyzing and critiquing arguments based on such incorrect use.

Standards for the Secondary Level (9 - 12)

Standard 3: Functions

The mathematical preparation of teachers of grades 9-12 must include experiences in which they:

- use the concept of function in the study of mathematics and the sciences.
- represent functions as symbolic expressions, verbal descriptions, tables, and graphs and move from one representation ot another;
- use the language of functions to describe and model change;
- investigate and discuss a variety of uses of functions in mathematics, business, and the physical bilogical, behavioral, and social sciences.

Standard 4: Probability, Statistics and Data Analysis

The mathematical preparation of teachers of graded 9-12 must include experiences in which they:

- collect, display, analyze, and interpret sample data in a variety of situations;
- use experimental and theoretical probabilities as appropriate to formulate and solve problems involving uncertainty.
- explore the probabilistic nature of statistical analyses including hypothesis testing, correlation, analysis of variance, and same nonparametric methods;
- investigate the role of estimation and probability in statistical analysis, including various methods of estimating parameters and errors;
- develop strategies for reasoning and making decisions based on uncertainty.

Standards for the Middle Grades (5 - 8)

Standard 5: Concept of Calculus
The mathematical preparation of middle grades teachers must provide experiences in which they:

- interpret, with the aid of graphs, diagrams, and physical models, the concepts of limit, differentiation and integrations, and the relationship among them;
- construct concrete examples of finite sequences, extend the ideas to infinite sequences and series, relating them to the meaning of approximation of nonterminating decimals and to the approximation of functions;
- explore concrete realistic problems involving average and instantaneous rates of change, areas, volumes, and curve lengths, and relate these problems to the concepts of differentiation and integration.

Standards for the Secondary Level (9 - 12)

Standard 5: Continuous Change
The mathematical preparation of teachers of grades 9—12 must provie experiences in which they:

- use properties and techniques of calculus to model phenomena in diverse settings;
- investigate the phenomenon of change as a limiting process;
- explore both intuitively and in depth the concepts of limit, continuity, differentiation, integration and other continuous processes;
- become familiar with the use of calculators with graphics capability and computer algebra systems both in the study and the applications of calculus.

Standard 6: Discrete Processes
The mathematical preparation of teachers of grades 9—12 must provie experiences in which they:

- investigate a variety of problem situations which lead to diverse discret mathematical models;
- develop and use a vaariety of counting techniques and counting arguments and their applications;
- gain experiences in algorithmic and recursive thinking, and develop skills in using algorithms and iterative and recursive techniques in solfing problems;
- learn to use appropriate technology effectively, including dealing with questions of computational efficiency and complexity;
- explore problems which involve the processes of scientific and socialdecision making.

Standard 7: Mathematical Structures
The mathematical preparation of teachers of grades 9—12 must provie experiences in which they:

- use mathematical structures which arise in the mathematical modeling of problems situations to explore and solve the problems;
- investigate mathematical structures that univy the observed patterns and properties that are shared by several diverse concrete examples;
- relate properties derived logically from the defining characteristics of a mathematical structure to properties of specific examples of the structure;
- explore the processes involved in building new structures from given structures (e.g., substructures, product structures, and quotient structures) and investigate properties and uses of such structures.

Appendix 2: Clarifying Roles in Systemic Change

The lists which follow are suggested as a basis for discussions to define and clarify the contributions each group might make to this important effort.

Teachers

Teachers are key figures in changing the way mathematics and science are taught and learned. As discussed in greater detail below, opportunities for professional development must be made available to all teachers if the Frameworks are to be implemented successfully. Making professional development a part of the school day and hiring teachers as leaders in mathematics and science will help make the Frameworks' vision a reality. In addition, teachers need to take increased ownership for their own development as professional educators.

Appropriate role activities for teachers can be considered within three broad categories.

Professional development

- Improve their own mathematics and science knowledge.
- Seek out opportunities for professional growth.
- Attend workshops and conferences.
- Join professional organizations.
- Continue to be lifelong learners.
- Read research and other professional publications.
- Participate in workshops and courses that cover content and pedagogy.

Teaching

- Hold high expectations for all their students.
- Notice their students strengths and build upon them.
- Elicit their students interests, and incorporate them into the curriculum.
- Make use of hands-on experiences and a variety of other instructional techniques.
- Meet with other teachers to reflect upon their own teaching and learning.
- Reflect upon educational issues and experiences.
- Try new instructional approaches.
- Plan lessons suitable for the diversity of students in the classroom.

Participation in and contribution to the school community

- Demonstrate leadership in their own schools and/or districts.
- Participate in school-based instructional support teams.
- Form study groups within schools.
- Model good teaching and learning.
- Join school-based committees.
- Collaborate with colleagues.
- Help make positive changes.
- Apply for grants to support needed programs.

Principals

Schools in which principals work with staff to adopt a plan of action that allows for experimentation and risk-taking make the implementation of new curricula much easier. Principals can support teachers who take on new roles and responsibilities by creating environments conducive to change. The school is an appropriate target area for changes in education, and it is the principal who is the key element for adopting and using new practices in a school.

Appropriate role activities for principals can be considered within three broad categories.

Professional development

- Become familiar with the Massachusetts Education Reform Act and the Curriculum Frameworks.
- Be aware of the current research on effective staff development.
- Support and participate in professional development activities.
- Keep current on district-adopted curriculum materials.
- Become knowledgeable about national standards and recognize the components of the Frameworks.
- Participate in the professional development process at the district and state levels.

Attitudes and behaviors in the school

- Encourage a positive, supportive climate that is conducive to change and teacher growth.
- Recognize teachers for professional work.
- Write letters of commendations for teachers who take on extra leadership responsibilities.
- Place a priority on curriculum issues.
- Communicate and consult with teachers.
- Motivate staff and encourage them to take risks.

- Discuss the difficulties of change with staff: time /pain/etc.
- Supervise and evaluate staff using multiple strategies for improving instruction.
- Be a catalyst for reflective practice.
- Create a school-wide plan with teachers for professional development.
- Implement peer-coaching in the school as a tool for improving instruction.
- Report mathematics and science activities and success stories to the media.
- Sponsor Family Math and/or Family Science Nights.
- Be visible in the school — visit classrooms, teach.

Material support for teaching and change

- Identify teacher leaders who can support other teachers.
- Set aside money for substitute teachers so teachers can attend conferences or visit other schools.
- Fund professional activities such as workshops and conferences.
- Offer release time to teachers to complete professional activities.
- Establish incentives for teachers who use new instructional techniques.
- Provide materials necessary for change.
- Make staff development part of the teachers' work role.
- Identify local resources (mentors, specialists, college faculty, parents, people from businesses, etc.) who can contribute to professional development programs.
- Support professional development in terms of time, money, resources, and materials.
- Use a variety of resources for the evaluation of staff.
- Apply for additional grant money to support needed programs.

Parents

As discussed in Chapter 4, parents are important resources for schools. Appropriate activities include the following.

- Be learners with their children.
- Participate in educational endeavors.
- Attend family/parent math and science programs at school.
- Demand equity of funds.
- Take responsibility for their children's education.

Students

Ultimately, the primary beneficiaries of improved mathematics and science teaching are the students in our schools. They, too, have an essential contribution to make.

- Study hard to learn and master skills
- Take responsibility for their own learning
- Reflect on what they are learning.

District level administrators and staff

The school district has the responsibility of ensuring that the goals for professional development stated in the Massachusetts Education Reform Act and the Massachusetts Curriculum Frameworks are met. The district therefore plays a major role in providing the resources necessary for the professional development of educational staff.

Appropriate role activities for district administrators and staff can be considered within three broad categories.

District policies and structure

- Restructure the school day to accommodate professional development activities so they don't remain as add-ons.
- Reward and recognize teachers who assume leadership roles and responsibilities.
- Hold principals and teachers accountable for ongoing professional development and lifelong learning. Link individual professional development plans for district math and science teachers to the district and state goals of implementing these frameworks.
- Revise assessment measures so that they are aligned with the proposed change.
- Reexamine the teacher evaluation process.
- Establish outreach to the community to support quality mathematics and science programs.

Material resources

- Provide time to staff to participate in professional development opportunities.
- Improve and continue to provide in-service programming.
- Create a professional development resource center from which teachers and principals may borrow materials.
- Funnel more money to schools to support professional development action plans.
- Establish professional development schools.
- Supply resources that support in-service programs.

- Create partnerships with cultural institutions, businesses, and universities.
- Provide in-service to support the implementation of the Massachusetts Curriculum Frameworks.
- Allow for teacher sabbaticals.
- Work to optimize class sizes.

Human resources

- Hire mathematics and science specialists who can make classroom visits and offer support in terms of ideas, time, resources, etc.
- Help schools identify needs and find ways to meet those needs.
- Implement mentor programs for beginning teachers.
- Hire quality mathematics and science teachers.

School Committees

Real change and improvement in mathematics and science education require sustained thought and action by the entire community. This process will be guided and supported by school committees throughout the commonwealth. Appropriate activities include the following.

- Study the frameworks and standards.
- Hire superintendents and other administrators who will provide instructional leadership for implementation.
- Provide funding for sound programs.
- Assure that the district provides meaningful in-service activities.
- Provide support for implementation.
- Visit classrooms and schools in order to understand, first hand, the new teaching methods, programs, and equipment being used.
- Support educational decisions.
- Provide resources and funding for programs.

School Councils

School councils at each school building have potential to make major contributions to school improvement. Appropriate activities could include:

- Study the frameworks and standards.
- Assure that the district provides meaningful in-service activities.
- Provide support for implementation.
- Visit classrooms and schools in order to understand, first hand, the new teaching methods, programs, and equipment being used.
- Support educational decisions.
- Focus the School Improvement Plan on implementation of the Frameworks.

Appendix 3: Chart for Planning and Evaluating Progress in Systemic Change

Determine existing local conditions and potential resources	Establish goals and create an action plan to meet them	Continuously evaluate progress on aspects of the plan
Continue to assess conditions and progress, and revise the plan		

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Massachusetts Curriculum Frameworks in Mathematics and in Science & Technology

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Curriculum Frameworks Response Form

Please take time to review this document and provide us with your comments. Your responses are very important and will help to direct the revisions which will be made by the framework revision committee. Please write your comments on this form. Attach a separate sheet if needed. **Return your comments by January 9, 1995 to:**

PALMS

Attn: Peg Bondorew and Mike Zapantis
Massachusetts Department of Education

350 Main Street • Malden, MA 02148

I. Reviewer Profile

Please supply the following information in the space provided.

Name (optional) or institution or group ________________________________

Address (optional)____

____

Date of Draft ______________________________

Note: If this response form is from a group or organization, please indicate the number represented in each category below. Individuals, please check appropriate categories.

Gender: ____Female ____Male

Race/Ethnicity: ____African American ____White ____Latino ____Native American
____Asian/Pacific Islander ____Other (please specify) ____________

____PreK - Gr. 4 Teacher ____Principal ____Parent

____Gr. 5 - 8 Teacher ____Math/Science Supervisor ____Community Member

____Gr. 9 - 12 Teacher ____District Supervisor ____Business

____ABE Teacher ____University Instructor ____Museum Educator

____Other:________________________________

II. Framework reviewed: ____Mathematics ____Science and Technology

Introduction: A Vision of Mathematics and Science Education

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about the introduction are:

My suggestions for improvement are:

Chapter 1: Critical Issues of the Framework

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 2: Teaching, Learning, and Assessing Mathematics/Science

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 3: Mathematics or Science and Technology (depending on Framework)

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Chapter 4: Implementing the Framework

I read this section ____not at all ____quickly ____carefully

My overall reaction is ____great ____okay ____has problems

Things I like about this chapter are:

My suggestions for improvement are:

Entire Framework

I read about _____ % of the document carefully. My overall reaction is:

Things I like about the draft:

My suggestions for improvement are:

Other Comments:

Request for Vignettes

In order to share rich instructional practices and bring to life the content, pedagogy and partnerships called for in the Curriculum Frameworks in Mathematics and in Science and Technology, the PALMS Curriculum Framework Committee continues to seek vignettes. Many of the vignettes which have been incorporated in this Review Draft were received as a result of earlier requests. In reading this Draft, you will find indications of the need for specific vignettes, and you may also see places where a vignette or example is not called for, but would fit nicely. Perhaps you have a vignette, or an idea for one, which you think belongs in the Frameworks, but you are not sure where it should be placed. We welcome all of your ideas.

Tips on Writing Vignettes

- Describe a classroom experience (or slight modification of a single event or series) to illustrate recommended instructional practices. Tell how students learned important mathematics and science. Share personal insight gained through this experience.
- Include actual dialogue between teacher and student or student and students. Make your story as believable as possible without distorting facts.
- Explain some of the important mathematical ideas that are imbedded in the vignette. The examples may incorporate content from more than one standard or discipline, illustrate how the students encountered the content, how they shared and grappled with misperceptions, and/or how they were able to derive some generalizations. Highlight teacher support and guidance.
- Include what the teacher was doing and thinking during the lesson. Why did the teacher make certain instructional decisions? How did the teacher interact with the students? What kinds of assessment information was gathered?
- As much as possible, describe experiences that include diverse groups of students and address issues of equity.

Vignettes may include one or more of the following elements:

- teacher as facilitator
- cooperative learning
- projects/investigation
- family mathematics/science
- student-to-student interactions
- use of manipulative materials
- learning styles
- use of technology
- community relations
- assessment
- relationship to National and Massachusetts Standards
- interdisciplinary learning

Mail vignettes to: PALMS
c/o Peg Bondorew/Mike Zapantis
Massachusetts Department of Education
350 Main Street • Malden, MA 02148-5023

For further information, please call (617)388-3300 extension 303